Solving challenging Math problems

Using Mathematica

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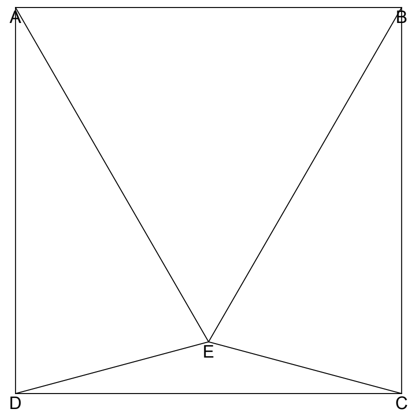
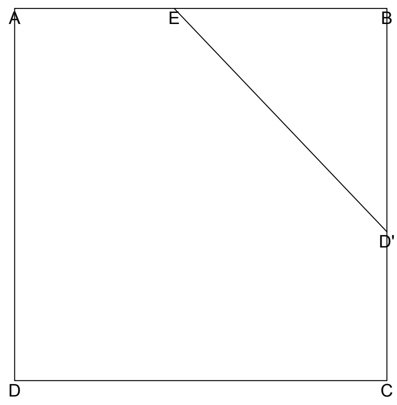
Problems

1. Given square ABCD with $\measuredangle EDC = \measuredangle ECD = 15°$, prove $\triangle ABE$ is equilateral.



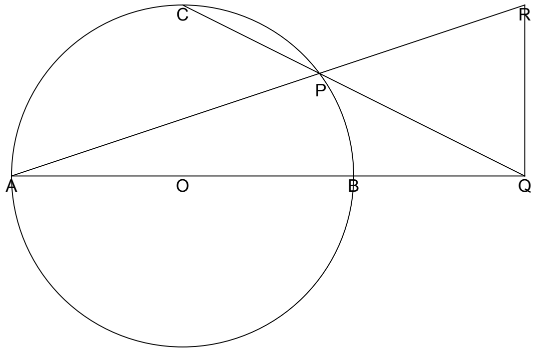
2. A square sheet ABCD is folded by placing the corner D at a point D’ on BC. Then, AD is moved to A’D’ which intersects AB in E. Prove that the circumference of EBD’ is half as long as the circumference of the square.

- Source: Nordic Mathematical Contest, 1994

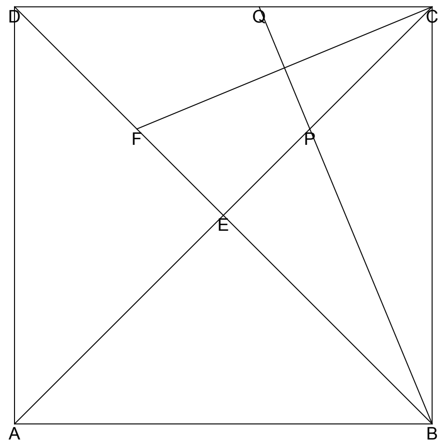


3. Let AB be the diameter of a circle with centre O. Pick a point C on the circle so that OC is perpendicular to AB. Let P be any point on the circle between C and B, and let the lines CP and AB intersect in Q. Choose R on AP such that RQ and AB are perpendicular to each other. Prove that $|BQ| = |QR|$.

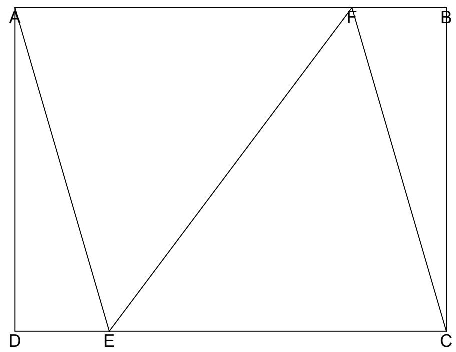
- Source: Nordic Mathematical Contest, 1994



4. Given that ABCD is a square, CF bisects ∡ACD, and BPQ is perpendicular to CF, prove that DQ = 2PE.

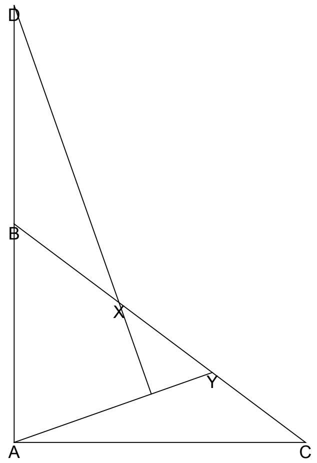


5. On sides AB and DC of rectangle ABCD, points F and E are chosen so that AFCE is a rhombus. If AB = 16 and BC = 12, find EF.



6. Let R be the region consisting of the points (x,y) of the cartesian plane satisfying both $|x| - |y| \leq 1$ and $|y| \leq 1$. Sketch the region R and find its area.
   - Source: Putnam, 1988
7. A dart, thrown at random, hits a square target. Assuming that any two parts of the target of equal area are equally likely to be hit, find the probability that the point hit is nearer to the center than to any edge. Express your answer in the form $(a\sqrt{b} + c)/d$, where a, b, c, d are positive integers.
8. Let s be any arc of the unit circle lying entirely in the first quadrant. Let A be the area of the region lying below s and above the x-axis and let B be the area of the region lying to the right of the y-axis and to the left of s. Prove that A + B depends only on the arc length, and not on the position, of s.
   - Source: Putnam, 1998
9. A hexagon is inscribed in a circle with radius r. Two sides of hexagon have length 1, two have length 2, and two have length 3. Prove that r is a root of the equation $2r^3-7r-3=0$.
   - Source: 7-th Nordic Mathematical Contest, 1993
10. Suppose L is a fixed line, and A a fixed point not on L. Let k be a fixed nonzero real number. For P a point on L, let Q be a point on the line AP with $|AP|.|AQ| = k^2$. Determine the locus of Q as P varies along the line L.

11. Let ABC be a right-angled triangle with right-angle at A. Let X be the foot of the perpendicular from A to BC, and Y the mid-point of XC. Let AB be extended to D so that $|AB| = |BD|$. Prove that DX is perpendicular to AY.

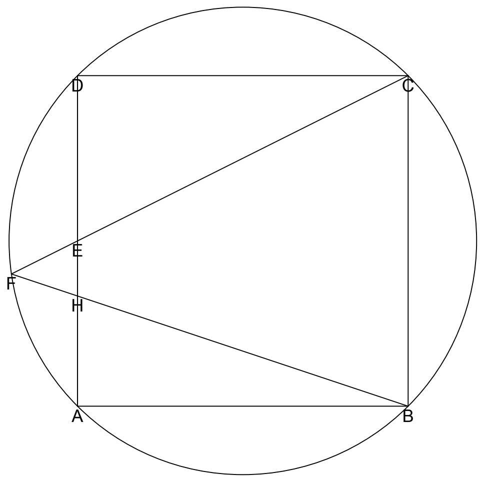


12. In the triangle ABC we have $|AB| = 1$ and $\angle ABC = 120°$. The perpendicular line to AB at B meets AC at D such that $|DC| = 1$. Find the length of AD.

- Source: 23rd Irish Mathematical Olympiad, 2010

13. The square ABCD is inscribed in a circle with centre O. Let E be the midpoint of AD. The line CE meets the circle again at F. The lines FB and AD meet at H. Prove $|HD| = 2|AH|$.

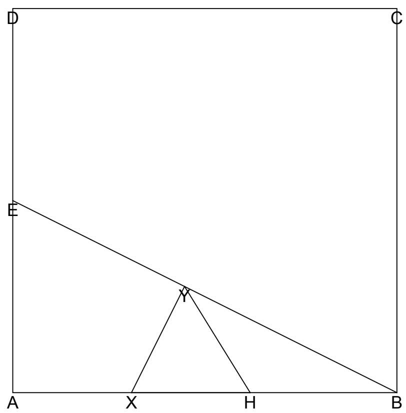
- Source: 27th Irish Mathematical Olympiad, 2014



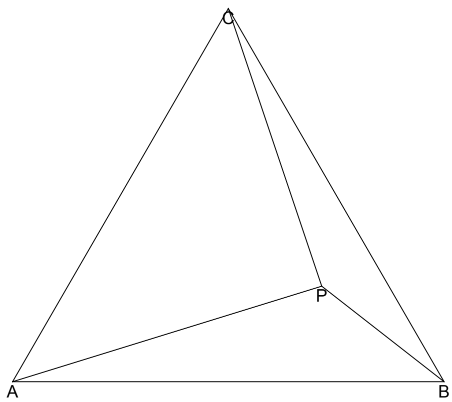
14. ABCD is a rectangle. E is a point on AB between A and B, and F is a point on AD between A and D. The area of the triangle EBC is 16, the area of the triangle EAF is 12 and the area of the triangle FDC is 30. Find the area of the triangle EFC.

15. Let ABCD be a square. The line segment AB is divided internally at H so that $|AB|.|BH| = |AH|2$. Let E be the mid point of AD and X be the midpoint of AH. Let Y be a point on EB such that XY is perpendicular to BE. Prove that $|XY| = |XH|$.

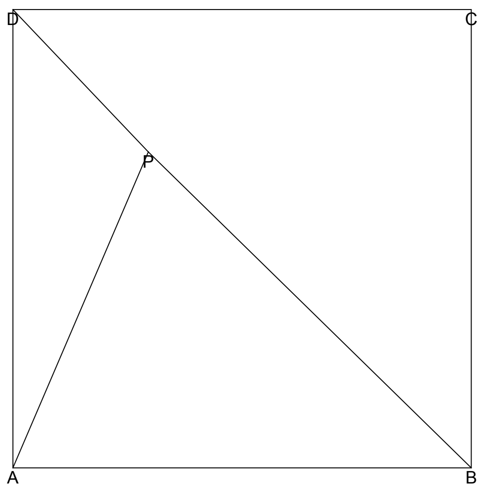
- Source: 22nd Irish Mathematical Olympiad, 2009



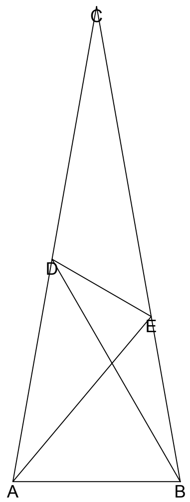
16. P is a point inside an equilateral triangle such that the distances from P to the three vertices are 3,
4 and 5, respectively. Find the area of the triangle.



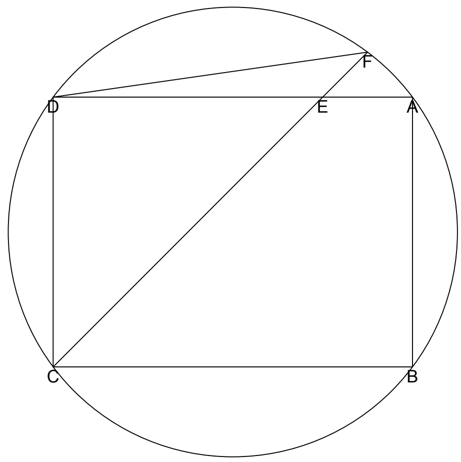
17. Find the area of a square ABCD containing a point P such that PA= 3,PB= 7,andPD= 5.



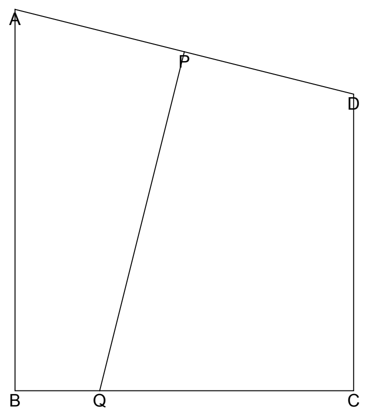
18. △ABC is isosceles with CA = CB. ∡ ABD = 60, ∡BAE = 50, and ∡C = 20. Find the measure of ∡EDB.



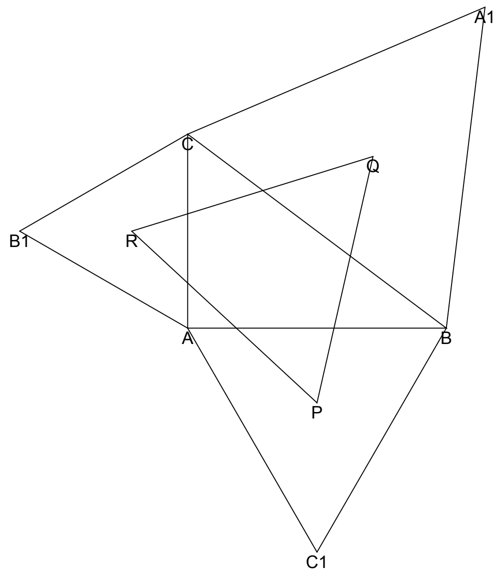
19. E is a point on side AD of rectangle ABCD, so that DE = 6, while DA = 8, and DC = 6. If CE extended meets the cir- cumcircle of the rectangle at F, find the measure of chord DF.



20. PQ is the perpendicular bisector of A D, AB is perpendicular to BC, and DC is perpendicular to BC. If AB = 9, BC = 8, and DC = 7, find the area of quadrilateral APQB.



21. (Napolean's Triangle) Given triangle ABC, construct an equilateral triangle on the outside of each of the sides. Let P, Q, R be the centroids of these equilateral triangles, prove that triangle P QR is equilateral.



22. Let $A_0, A_1, A_2, A_3,\ A_4$ divide a unit circle into five equal parts. Prove that the chords $A_0A_1,\ A_0A_2$ satisfy

$(A_0A_1 \,.\, A_0A_2)^2 = 5$

23. Let $A_0, A_1, A_2, A_3, A_4, A_5, A_6$ be a regular 7-gon. Prove that

$$\frac{1}{A_0\,A_1} + \frac{1}{A_0\,A_2} = \frac{1}{A_0\,A_3}$$

24. If ABCD is a parallelogram, prove that

$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$.

25. Let G denote the centroid of triangle ABC. Prove that

$3(GA^2 + GB^2 + GC^2) = AB^2 + BC^2 + CA^2$.

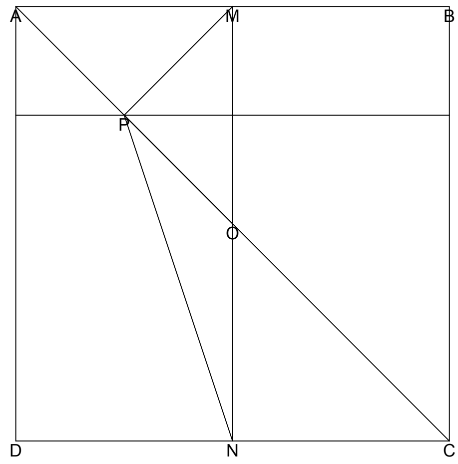
26. Equilateral triangles whose sides are 1,3,5,7...are placed so that the bases lie corner along a straight line.Show that the vertices lie on a parabola and are all at integral distances from its focus.

27. Find a relation that must hold between the parameters a, b, c so that the line $\frac{x}{a} + \frac{y}{b} = 1$ will be tangent to the circle $x^2 + y^2 = c^2$.

28. A parabola with equation $y^2 = ax$ is cut in four points by the circle $(x - h)^2 + (y - k)^2 = r^2$. Determine the product of the distances of the four points of intersection from the axis of the parabola.

29. Let ABCD be a square of center O. The parallel through O to AD intersects AB and CD at the points M and N, respectively, and a parallel to AB intersects the diagonal AC at the point P . Prove that

$OP^4 + \left(\frac{MN}{2}\right)^4 = MP^2 . NP^2$



30. Find the minimum value of

$(u-v)^2 + \left(\sqrt{2-u^2} - \frac{9}{v}\right)^2$ for $0 < u < \sqrt{2}$ and $v > 0$

- Source: Putnam, 1984

31. Let V be the region in the cartesian plane consisting of all points (x,y) satisfying the simultaneous conditions

$|x| \le y \le |x| + 3$ and $y \le 4$. Find the centroid of V.

- Source: Putnam, 1982

32. Determine the minimum value of

$(r-1)^2+\left(\frac{s}{r}-1\right)^2+\left(\frac{t}{s}-1\right)^2+\left(\frac{4}{t}-1\right)^2$

for all real numbers r,s,t with $1 \le r \le s \le t \le 4$.

33. Find the area of a convex octagon that is inscribed in a circle and has four consecutive sides of length 3 units and remaining 4 consecutive sides of length 2 units. Give the answer in the form of $r + s\sqrt{t}$ with r,s and t positive integers.

34.

35. The sum of the first n terms of the sequence $1, (1+2), (1+2+2^2),\ldots,(1+2+\cdots+2^{k-1}),\ldots$ is of the form $2^{n+R} + Sn^2 + Tn + U$ for all $n > 0$. Find R,S,T and U.

36. Given the linear fractional transformation of x into $f_1(x)=(2x-1)/(x+1)$, define $f_{n+1}(x)=f_1(f_n(x))$ for n=1,2,3,.... It can be shown that $f_{35} = f_5$. Determine A,B,C, and D so that $f_{28}(x) = (Ax+B)/(Cx+D)$.

37. The number $2^{48} - 1$ is exactly divisible by what two numbers between 60 and 70?

38. What is the remainder when $X^{1982}+1$ is divided by $X - 1$? Verify your answer.

39. A sequence $\{u_n\}$, n=0,1,2,..., is defined by $u_0 = 5$, $u_{n+1} = u_n + n^2 + 3n + 3$, for n = 0,1,2,.... If u_n is expressed as a polynomial $u_n = \sum_{k=0}^{d} c_k n^k$, where d is the degree of the polynomial, find the sum $\sum_{k=0}^{d} c_k$.

40. Find $\sum_{n=2}^{\infty} \frac{n^2-2n-4}{n^4+4n^2+16}$.

41. Evaluate $\int_0^2 \frac{(16-x^2)\,x}{16-x^2+\sqrt{(4-x)(4+x)(12+x^2)}}\,dx$

42. Let X= $\begin{pmatrix} 7 & 8 & 9 \\ 8 & -9 & -7 \\ -7 & -7 & 9 \end{pmatrix}$, Y = $\begin{pmatrix} 9 & 8 & -9 \\ 8 & -7 & 7 \\ 7 & 9 & 8 \end{pmatrix}$, let A = Y^{-1}- X, and let B be the inverse of $X^{-1} + A^{-1}$. Find a matrix M such that M^2 = XY −BY.

43. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos^4(x)+\sin(x)\cos^3(x)+\sin^2(x)\cos^2(x)+\sin^3(x)\cos(x)}{\sin^4(x)+\cos^4(x)+2\sin(x)\cos^3(x)+2\sin^2(x)\cos^2(x)+2\sin^3(x)\cos(x)}\, dx$

44. Find $\sum_{i=1}^{\infty} \frac{i^2-2}{(i+2)!}$

45. Given that 40! = abc def 283 247 897 734 345 611 269 596 115 894 272 pqr stu vwx.find p,q,r,s,t,u,v,w,x, and then find a,b,c,d,e, f.

46. How many sequences of 1's and 3's sum to 16? (Examples of such se- quences are {1,3,3,3,3,3} and {1,3,1,3,1,3,1,3}.)

47. Evaluate $\int_1^4 \frac{x-2}{(x^2+4)\sqrt{x}}\, dx$

48. Show that $\int_0^{\frac{\pi}{4}} \frac{1}{2+\mathrm{Tan}[\theta]}\, d\theta = \frac{\pi+\ln(9/8)}{10}$

49. Solve the initial value problem $\frac{dy}{dx}$ = yln y + ye^x , y(0) = 1 (i.e. find y in terms of x).

50. Recall that the Fibonacci numbers F(n) are defined by F(0) = 0, F(1) = 1, and F(n) = F(n−1)+F(n−2) for n ≥ 2. Determine the last digit of F(2006) (e.g. the last digit of 2006 is 6).

51. Compute $\int_0^1 \left((e-1)\sqrt{\mathrm{Log}[1+e\,x-x]} + e^{x^2} \right)$

52. It is known that $2\cos^3\left(\frac{\pi}{7}\right) - \cos^2\left(\frac{\pi}{7}\right) - \cos\left(\frac{\pi}{7}\right)$ is a rational number. Write this rational number in the form p/q, where p and q are integers with q positive.

53. Find rational numbers a,b,c,d,e such that
$\sqrt{7+\sqrt{40}} = a + b\sqrt{2} + c\sqrt{5} + d\sqrt{7} + e\sqrt{10}$

54. How many different (i.e. pairwise non-congruent) triangles are there with integer sides and with perimeter 1994?

55. Find the first integer n > 1 such that the average of
$1^2, 2^2, 3^2 \ldots, n^2$ is itself a perfect square.

56. Find, showing your method, a six-digit integer n with the following properties:
(i) n is a perfect square,
(ii) the number formed by the last three digits of n is exactly one greater than the number formed by the first three digits of n.

57. Find the number of polynomials of degree 5 with distinct coefficients from the set {1, 2, 3, 4, 5, 6, 7, 8} that are divisible by x^2 - x + 1.

58. Prove that the number $512^3 + 675^3 + 720^3$ is composite.

59. Find four positive integers, each not exceeding 70000 and each having more than 100 divisors.

60. Determine all three-digit numbers N having the property that N is divisible by 11, and $\frac{N}{11}$ is equal to the sum of the squares of the digits of N.

61. Let p and q be natural numbers such that
$\frac{p}{q} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots - \frac{1}{1318} + \frac{1}{1319}$
Prove that p is divisible by 1979.

62. Show that for every natural number n the product
$(4-\frac{2}{1})(4-\frac{2}{2})(4-\frac{2}{3})\ldots(4-\frac{2}{n})$ is an integer.

63. Find an integer n, where 100 ≤ n ≤ 1997, such that $\frac{2^n+2}{n}$ is also an integer.

64. Find the sum of all distinct positive divisors of the number 104060401.

65. Prove that 1280000401 is composite.

66. Find the factor of $2^{33} - 2^{19} - 2^{17} - 1$ that lies between 1000 and 5000.

67. Represent the number 989·1001·1007+ 320 as the product of primes.

68. One of Euler's conjecture was disproved in the 1980s by three American Mathematicianswhen they showed that there is a positive integer n such that
$n^5 = 133^5 + 110^5 + 84^5 + 27^5$.
Find the value of n.

69. The number 21982145917308330487013369 is the thirteenth power of a positive integer. Which positive integer?

70. Determine all pairs (x, y) of positive integers satisfying the equation $(x + y)^2 - 2(xy)^2 = 1$.

71. Are there integers m and n such that $5m^2 - 6mn + 7n^2 = 1985$?

72. Show that the only solutions of the equation $x^3 - 3xy^2 - y^3 = 1$ are given by (x, y) = (1, 0),(0, −1),(−1, 1),(1, −3),(−3, 2),(2, 1).

73. Show that the polynomial $x^8 + 98x^4 + 1$ can be expressed as the product of two nonconstant polynomials with integer coefficients.

74. Determine the maximum value of m^2+n^2, where m and n are integers satisfying m, n ∈ {1, 2, ..., 1981} and $(n^2 - mn - m^2)^2=1$

- IMO 1981/3

75. It is given that 2^{333} is a 101-digit number whose first digit is 1. How many of the numbers 2^k, 1 ≤ k ≤ 332, have first digit 4?

- Tournament of the Towns Fall Junior-A/7

76. Let $x_1 = x_2 = 1$, and $x_{n+1} = 1996\, x_n + 1997\, x_{n-1}$ for $n \geq 2$. Find the remainder of x_{1997} upon division by 3.

77. Evaluate $\sum_{k=1}^{n} 2^{k-1}$ for $n = 1, 2, \ldots$

78. A sequence $a_0, a_1, a_2 \ldots$ of real numbers is defined recursively by $a_0 = 1$, $a_{n+1}= \frac{a_n}{1+na_n}$ (n = 0,1,2,...). Find the general formula for a_n.

79. Let $a_n$denote the integer closest to $\sqrt{n}$. Evaluate the sum
$S = \frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\ldots+\frac{1}{a_{1980}}$

80. Let $N = 9 + 99 + 999 + \ldots + 99\overbrace{\ldots}^{99}9$. Determine the sum of digits of N.

81. Evaluate $\frac{1}{1\cdot2\cdot3} + \frac{1}{3\cdot4\cdot5}+\frac{1}{5\cdot6\cdot7}+\ldots$

82. $\lim_{n\to\infty} \frac{1}{n^2}\prod_{i=1}^{n} \left(n^2 + i^2\right)^{1/n}$

83. Let $F_n$denote the Fibonacci sequence. Evaluate $\sum_{k=1}^{\infty} \frac{F_k}{3^k}$.

84. Define a sequence $\{a_n\}$ by $a_0 = 0$, $a_1 = 1$, $a_2=2$, and for n ≥ 3, $a_n=a_{n-1}+a_{n-2}-a_{n-3}+1$.Find a_n.

85. Let $f(n) = \sum_{k=1}^{n} \left[\frac{n}{k}\right]$ where [x] denotes the greatest integer ≤x, and let $g(n) = (-1)^{f(n)}$.Find g(2004).

86. $\int_0^{\pi} \ln(\sin x)\, dx$

87. For any positive integer n, define a sequence $\{n_k\}$as follows: Set n_0 = n, and for each k ≥ 1, let n_k be the sum of the (decimal) digits of n_{k-1}. For example, for n = 1729 we get the sequence 1729, 19, 10, 1, 1, 1, ... In general, for any given starting value n, the resulting sequence $\{n_k\}$ eventually stabilizes at a single digit value. Let f(n) denote this value; for example, f(1729) = 1. Determine f(2^{2006}).

88. Let f(n) = $(1^2 + 1)1! + (2^2 + 1)2! + \cdots + (n^2 + 1)n!$.
Find a simple general formula for f(n).

89. For any positive integer k let f_1(k) denote the sum of the squares of the digits of k (when written in decimal), and for n ≥ 2 define f_n(k) iteratively by f_n(k) = $f_1(f_{n-1}$(k)). Find f_{2007}(2006).

90. Prove that the sum of the infinite series $\sum_{n=1}^{\infty} \frac{n^{2009}}{2^n}$ is an integer.

91. Let $H_n = \sum_{k=1}^{n} \frac{1}{n}$. Find the value of $\sum_{n=1}^{\infty} \frac{H_{n+1}}{n(n+1)}$.

92. Let f(x) = $\frac{x-1}{x+1}$,and let f_k(x) be the k-th iterate of f(x) defined by f_1(x) = f(x) and f_k(x) = f(f_{k-1}(x)) for k = 2, 3, 4, ... Find, with proof, f_{2015}(2015).

93. Evaluate the integral$\int_0^{\infty} \frac{\log(x)}{x^2+9}\, dx$

94. Let f(n) = $\sum_{k=1}^{n} \frac{1}{\sqrt{k+1} + \sqrt{k}}$. Evaluate f(9999).

95. Let f(n) denote the n-th term in the sequence 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, . . . , obtained by writing one 1, two 2's, three 3's, four 4's, etc. Find f(2013)

96. Evaluate $\sum_{k=n}^{2n} \binom{k}{n} 2^{2n-k}$.

97. Determine, with proof, the rightmost digit (in decimal) of $\langle \frac{10^{20000}}{10^{100}+3} \rangle$ (where ⟨x⟩ denotes the largest integer ≤ x).

98. Find a simple formula for the sum $\sum_{k=1}^{n} \frac{k}{(k+1)!}$.

99. Find a simple formula for the sum $\sum_{k=0}^{n} 2^{-k} \binom{k+n}{k}$

100. Given a positive integer n, let n1 be the sum of digits (in decimal) of n, n2 the sum of digits of n1, n3 the sum of digits of n2, etc. The sequence {ni} eventually becomes constant, and equal to a single digit number. Call this number f(n). For example, f(1999) = 1 since for n = 1999, n1 = 28, n2 = 10, n3 = n4 = · · · = 1. How many positive integers n ≤ 2001 are there for which f(n) = 9?

101. Find all x such that $\sum_{k=1}^{\infty} k x^k = 20$.

102. A pair of positive integers is golden if they end in the same two digits. For example (139, 2739) and (350, 850) are golden pairs. What is the sum of all two-digit integers n for which (n^2,n^3) is golden?

103. Let ⟨x⟩ denote the floor function (the largest integer less than or equal to x). Find the average value
of the quantity $\langle 2x^3 - 2\langle x^3 \rangle \rangle$ on the interval $(-\frac{3}{2}, \frac{3}{2})$.

104. You have a 10×10 grid of squares. You write a number in each square as follows: you write 1, 2, 3, . . . , 10 from left to right across the top row, then 11, 12, . . . , 20 across the second row, and so on, ending with a 100 in the bottom right square. You then write a second number in each square, writing 1, 2, . . . , 10 in the first column (from top to bottom), then 11, 12, . . . , 20 in the second column, and so forth. When this process is finished, how many squares will have the property that their two numbers sum to 101?

105. Let S_n be the sum $S_n = 1 + 11 + 111 + 1111 + \ldots + 111\ldots 11$ where the last number $111\ldots 11$ has exactly n 1's. Find Floor[$\frac{10^{2017}}{S_{2014}}$].

106. Let f(x) = $x^2 - 2$, and let f^n denote the function f applied n times. Compute the remainderwhen $f^{24}(18)$ is divided by 89.

107. Let ω be a fixed circle with radius 1, and let BC be a fixed chord of ω such that BC = 1. The locus of the incenter of ABC as A varies along the circumference of ω bounds a region R in the plane.Find the area of R.

108. Let M denote the number of positive integers which divide 2014!, and let N be the integer closest to ln(M). Estimate the value of N.

109. Evaluate the infinite sum $\sum_{n=2}^{\infty} \log_2\left(\frac{1-\frac{1}{n}}{1-\frac{1}{n+1}}\right)$.

110. Simplify sin(100 °)+sin(1000 °)+sin(10000 °)+2 cos(10 °)

111. Simplify $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix}\ldots\begin{pmatrix} 1 & 99 \\ 0 & 1 \end{pmatrix}$.

112. Find all ordered 4-tuples (a,b,c,d) satisfying the following system of equations

$a^2 - b^2 - c^2 - d^2 = -b + c - 2$

$2\,a\,b = a - d - 32$

$2\,a\,c = -a - d + 28$

$2\,a\,d = b + c + 31$

113. The fraction $\frac{1}{2015}$ has a unique "restricted partial decomposition " of the form $\frac{1}{2015} = \frac{a}{5} + \frac{b}{13} + \frac{c}{31}$ where a, b, c are integers with $0 \le a < 5$, $0 \le b < 13$. Find $a + b$.

114. How many numbers between 1 and 1, 000, 000 are perfect squares but not perfect cubes?

115. How many numbers less than 1,000,000 are the product of exactly 2 distinct primes?

116. Let p(x) be the polynomial of degree 4 with roots 1, 2, 3, 4 and leading coefficient 1. Let q(x) be the polynomial of degree 4 with roots 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and leading coefficient 1. Find $\lim_{x\to 1} \frac{p(x)}{q(x)}$.

117. Evaluate the sum $\cos\left(\frac{2\pi}{18}\right) + \cos\left(\frac{4\pi}{18}\right) + \ldots + \cos\left(\frac{34\pi}{18}\right)$.

118. Johnny the grad student is typing all the integers from 1 to ∞, in order. The 2 on his computer is broken however, so he just skips any number with a 2. What's the 2008th number he types?

119. Determine the last two digits of 17^{17}, written in base 10.

120. Find the coefficient of x^6 in the expansion of $(x+1)^6 \sum_{i=0}^{6} x^i$.

121. What is the sum of all integers x such that |x + 2| ≤ 10?

122. What is the largest x such that x^2 divides $24 \cdot 35 \cdot 46 \cdot 57$?

123. John M. is sitting at (0, 0), looking across the aisle at his friends sitting at (i, j) for each 1 ≤ i ≤ 10 and 0 ≤ j ≤ 5. Unfortunately, John can only see a friend if the line connecting them doesn't pass through any other friend. How many friends can John see?

124. ABCDE is a regular pentagon inscribed in a circle of radius 1. What is the area of the set ofpoints inside the circle that are farther from A than they are from any other vertex?

125. Call an integer n > 1 radical if $2^n - 1$ is prime. What is the 20th smalles radical number?

126. \.10Find all solutions to $x^4 + 2\,x^3 + 2\,x^2 + 2\,x + 1 = 0$.

127. Find all real roots of $x^9 + \frac{9x^6}{8} + \frac{27x^3}{64} - x + \frac{219}{512} = 0$.

128. How many of the first 1000 positive integers can be written as the sum of finitely many distinct numbers from the sequence 3^0, 3^1,3^2, ...?

129. For a positive integer n, let τ (n) be the number of divisors of n. Determine $\sum_{n=1}^{2012} \tau(n)$.

130. FInd the remainder of the division of the polynomial
x+x^9+x^{25}+x^{49}+x^{81} by x^3- x.

131. An ant is leashed up to the corner of a solid square brick with side length 1 unit. The length of the ant's leash is 6 units, and it can only travel on the ground and not through or on the brick. What is the area of region accessible to the ant?

132. Let ABC be a triangle such that AB = 3, BC = 4, and AC = 5. Let X be a point in the triangle. Compute the minimal possible value of $AX^2 + BX^2 + CX^2$.

133. Compute the number of ways there are to select three distinct lattice points in three-dimensional space such that the three points are collinear and no point has a coordinate with absolute value exceeding 1.

134. For any positive integer x ≥ 2, define f(x) to be the product of the distinct prime factors of x. For example, f(12) = 2 · 3 = 6. Compute the number of integers 2 ≤ x < 100 such that f(x) < 10.

135. Let x be a two-digit positive integer. Let x' be the number achieved by switching the two digits in x (for example: if x = 24, x' = 42). Compute the number of x's that exist such that x + x' is a perfect square.

136. We say that a number is arithmetically sequenced if the digits, in order, form an arithmetic sequence. Compute the number of 4-digit positive integers which are arithmetically sequenced.

137. Let a 5 digit number be termed a "valley" number if the digits (not necessarily distinct) in the number a b c d e satisfy a > b > c and c < d < e. Compute the number of valley numbers that start with 3.

138. Compute the smallest positive integer with exactly 6 distinct factors.

139. O is a circle with radius 1. A and B are fixed points on the circle such that AB =$\sqrt{2}$. Let C be any point on the circle, and let M and N be the midpoints of AC and BC, respectively. As C travels around circle O, find the area of the locus of points on MN.

140. Let E be an ellipse with major axis length 4 and minor axis length 2. Inscribe an equilateral triangle ABC in E such that A lies on the minor axis and BC is parallel to the major axis.Compute the area of triangle ABC.

141. Compute the integral $\int_0^2 \left(\sqrt{\frac{4-x}{x}} - \sqrt{\frac{x}{4-x}}\right) dx$

142. Compute the sum $\sum_{i=1}^{\infty} \frac{1}{2^i\, i^2}$

143. The set of points (x, y) in the plane satisfying $x^{2/5}$+ | y | = 1 form a curve enclosing a region. Compute the area of this region.

144. Integrate $\int_{-2}^{2} \frac{x^2+1}{2^x+1} dx$

145. Compute $\lim_{x\to\infty} \left(x - x^2 \log\left(\frac{x+1}{x}\right)\right)$

146. Given the digits 1 through 7, one can form 7! = 5040 numbers by forming different permutations of the 7 digits (for example, 1234567 and 6321475 are two such permutations). If the 5040 numbers obtained are then placed in ascending order, what is the 2013th number?

147. Given a complex number z such that $z^{13} = 1$, find all possible values of $z+z^3+z^4+z^9+z^{10}+z^{12}$.

148. Find the sum of all real x such that
$\frac{4x^2+15x+17}{x^2+4x+12} = \frac{5x^2+16x+18}{2x^2+5x+13}$

149. Compute the largest root of $x^4 - x^3 - 5x^2 + 2x + 6 = 0$

150. Find all real x that satisfy $\sqrt[3]{20x + \sqrt[3]{20x+13}} = 13$

151. Let $a = -\sqrt{3} + \sqrt{5} + \sqrt{7}$, $b = \sqrt{3} - \sqrt{5} + \sqrt{7}$, $c = \sqrt{3} + \sqrt{5} - \sqrt{7}$
$\frac{a^4}{(a-b)(a-c)} + \frac{b^4}{(b-a)(b-c)} + \frac{c^4}{(c-a)(c-b)}$

152. A triangle has sides of length $\sqrt{2}$,3+$\sqrt{3}$,2$\sqrt{2}$+$\sqrt{6}$ Compute the area of the smallest regular polygon that has three vertices coinciding with the vertices of the given triangle.

153. Evaluate $\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{m\,n\,(m+n+1)}$.

154. Find the number of triples of nonnegative integers (x,y,z) such that 15x + 21y + 35z = 525.

155. Evaluate $\sum_{n=1}^{\infty} \frac{1}{n^3\,(n+1)^3}$.

156. Two different squares are randomly chosen from an 8 × 8 chessboard. What is the probabilitythat two queens placed on the two squares can attack each other? Recall that queens in chess can attack any square in a straight line vertically, horizontally, or diagonally from their current
position.

157. Evaluate $\sum_{n=1}^{\infty} \frac{(7\,n+32)\,3^n}{n\,(n+2)\,4^n}$

158. In a unit square ABCD, find the minimum of
$\sqrt{2}$ AP + BP + CP where P is a point inside ABCD.

159. Take a clay sphere of radius 13, and drill a circular hole of radius 5 through its center.Take the remaining “bead” and mold it into a new sphere. What is this sphere’s radius?

160. Find the area of the region of the xy-plane defined by the inequality $|x|+|y|+|x+y| \le 1$.

161. Given is a regular tetrahedron of volume 1. We obtain a second regular tetrahedron by reflecting the given one through its center. What is the volume of their intersection?

162. Find the volume of the region which is common to the interiors of the three circular cylinders $y^2 + z^2 = 1$, $z^2 + x^2 = 1$ and $x^2 + y^2 = 1$.

163. A rectangular box has sides of length 3, 4, 5. Find the volume of the region consisting of all points that are within distance 1 of at least one of the sides.

164. Evaluate $\int_0^{\infty} \frac{x^3}{e^x - 1}\, dx$

165. Evaluate $\int_{-1}^{1} \sqrt{\frac{x+1}{1-x}}\, dx$

166. Evaluate $\int_0^{\infty} \frac{\log(x)}{x^2+1}\, dx$

167. Evaluate $\int_0^{1} \frac{1}{(2\,x+3\,(1-x))^2}\, dx$

168. Evaluate $\int_0^{1} \frac{x^4\,(1-x)^4}{x^2+1}\, dx$

169. Solve
$2\,x^2 - 4\,x\,y + 3\,y^2 = 36$
$3\,x^2 - 4\,x\,y + 2\,y^2 = 36$

170. Solve
5732x + 2134y + 2134z = 7866
2134x + 5732y + 2134z = 670
2134x + 2134y + 5732z = 11464

171. Evaluate $\sum_{k=1}^{n} \frac{(-1)^k k}{4 k^2 - 1}$

172. Evaluate $\sum_{k=1}^{n} 4^{-k} \binom{2k}{k}$

173. Find a six-digit number whose product by 2,3,4,5 or 6 contains the same digits as did the original number.

174. The integer A consists of 666 threes, and the integer B has 666 sixes. What digits appear in the product A·B?

175. Adjoin to the digits 523... three more digits such that the resulting 6 digit number is divisible by 7,8 and 9.

176. Using all the digits from 1 to 9, make up three, three digit numbers which are related in the ration 1:2:3.

177. Find the sum of all the four-digit even numbers which can be written using 0,1,2,3,4,5.

178. All the integers beginning with 1 are written successively (that is, 1234567891011...).What digit occupies 206,788th position.

179. Evaluate $\sum_{i=1}^{n} \cos\left(\frac{2 \pi i}{2 n+1}\right)$

180. Evaluate $\sum_{k=0}^{n} \binom{n}{k}^2$

181. Is the coefficient of x^{20} greater in $\left(1 + x^2 - x^3\right)^{1000}$ or $\left(1 - x^2 + x^3\right)^{1000}$?

182. Prove that in the product $\left(1 - x + x^2 - x^3 + ... + x^{100}\right)\left(1 + x + x^2 + x^3 + ... + x^{100}\right)$ after multiplying and collecting the terms, there does not appear a term in x of odd degree.

183. Solve |x+1|-|x|+3 |x-1|-2 |x-2|=x+2 for x.

184. Solve $\sqrt{x - 4\sqrt{x-1} + 3} + \sqrt{x - 6\sqrt{x-1} + 8} = 1$ for x.

185. Factorise $a^{10} + a^5 + 1$.

186. Solve $\sqrt{a - \sqrt{a + x}} = x$.

187. A triangle has its lengths in an arithmetic progression, with difference d. The area of the triangle is t. Find the lengths and angles of the triangle.

188. Can 1986 be expressed as a sum of 6 odd squares?

189. Find all real solutions of the equations:
$(x + y)^3 = z$
$(y + z)^3 = x$
$(x + z)^3 = y$

190. What digit must be put in place of ? in the number 888...888?999...99 (where the 8 and 9 are each written 50 times) in order that the resulting number is divisible by 7?

191. 100 numbers 1,1/2,1/3,...,1/100 are written on the blackboard.One may delete two arbitrary numbers a and b among them and replace them with the number a+b+ab. After 99 such operations only one number is left. What is the final number?

192. 11 girls and n boys went for mushrooms.They have found n^2+ 9n-2 in total, and each child has found the same quantity.Which is greater:the number of girls or the number of boys?

193. Find a if and b are integers such that x^2-x-1 is a factor of $ax^{17}+ bx^{16}+1$.

194. Find all integers c such that x^2-x+c is a factor of x^{13}+ x+90.

195. Evaluate $\sum_{k=0}^{995} \frac{(-1)^k \binom{1991-k}{k}}{1991-k}$

196. If A=(0,-10) and B=(2,0), find the point(s) C on the parabola y=x^2 which minimizes the area of triangle ABC.

197. Integrate $\int (x^6 + x^3) \sqrt[3]{x^3 + 2}\, dx$

198. Let C be a circle with center 0, and Q a point inside C diferent from O. Show that the area enclosed by the locus of the centroid of triangle OPQ as P moves about the circumference of C is independent of Q.

199. Solve for all real x and y

$x^4 - 6 x^2 y^2 + y^4 = 1$

$4 x^3 y - 4 x y^3 = 1$

200. Evaluate $\sum_{i=1}^{\infty} \frac{1}{2 i^2 - i}$

201. Evaluate $\sum_{k=0}^{n} \frac{(-1)^k \binom{n}{k}}{2 k+2 n+1}$

202. Solve the equation

$\frac{1-\sqrt{2}+\sqrt{3}}{1+\sqrt{2}-\sqrt{3}} = \frac{1}{2}\left(\sqrt{x} + \sqrt{y}\right)$

203. Suppose P(x) is a polynomial of degree 8 with real coefficients and P(k) = $\frac{1}{k}$for k = 1,2,3..9. Determine the number P(10).

204. Show that $\sqrt{\text{PASSION}}$ = KISS has a unique solution in base 10. That is, find the unique correspondence between the letters P,A,S,I,O,N,K and seven of the digits 0,1,2,...9 such that the equation holds.

205. Provide an approximate value of the integral $\int_1^{100} x^x\, dx$.

206. What is the value of $a^4+b^4+c^4$ if

$a+b+c = 3$

$a^2+b^2+c^2 = 5$

$a^3+b^3+c^3 = 7$

207. Find integer solutions of the equations

$a+b+c = 3$

$a^3+b^3+c^3 = 3$

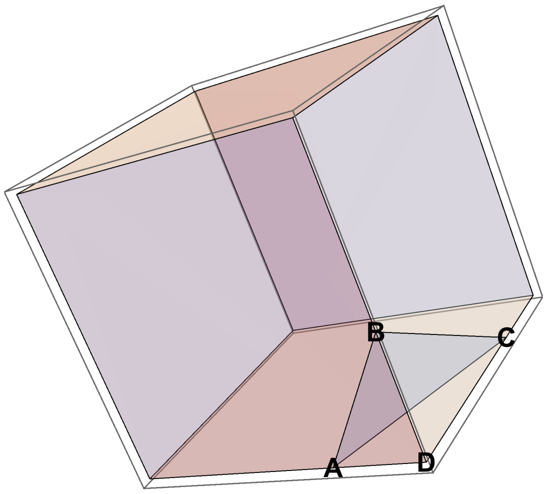
208. Is there a 4 digit number that is reversed by multiplication by 4?

209. Solve the following equation for the positive integers x and y: $(360 + 3x)^2$=492y04.

210. Prove that cot 10° cot 30° cot 50° cot 70°= 3

211. In 1996 nobody could claim that on their birthday their age was the sum of the digits of the year in which they were born. What was the last year prior to 1996 which had the same property?

212. As shown in the first figure below, a large wooden cube has one corner sawed off forming a tetrahedron ABCD. Determine the length of CD, if AD = 6, BD = 8 and area(Δ ABC) = 74.



213. Determine the number of points (x; y) on the hyperbola 2xy -5x + y = 55 for which both x and y are integers.

214. Let a,b,c,d be distinct real numbers such that
a+b+c+d =3 and $a^2+b^2+c^2+d^2= 45$
Find the value of the expression
$$\frac{a^5}{(a-b)\ (a-c)\ (a-d)}+\frac{b^5}{(b-a)\ (b-c)\ (b-d)}+\frac{c^5}{(c-a)\ (c-b)\ (c-d)}+\frac{d^5}{(d-a)\ (d-b)\ (d-c)}$$

215. Find all pairs of positive integers (m; n) for which m^2- n^2 =1995.

216. Express $\frac{19}{94}$in the form $\frac{1}{m}+\frac{1}{n}$, where m and n are positive integers.

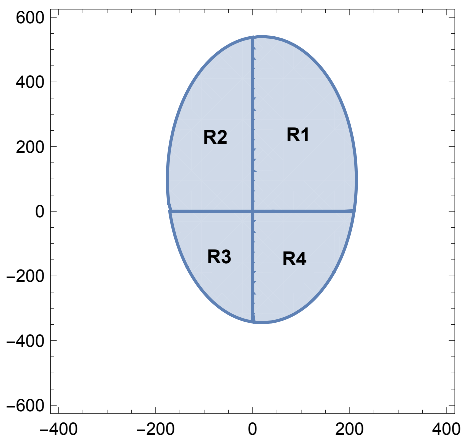
217. Find $x^2+y^2+z^2$, if x, y, and z are positive integers such that
$7x^2$-$3y^2$+$4z^2$=8 and $16x^2$-$7y^2$+$9z^2$= -3.

218. The sides of 4ABC measure 11, 20, and 21 units. We fold it along P Q, QR, and RP , where P , Q, and R are the midpoints of its sides,until A, B, and C coincide. What is the volume of the resulting tetrahedron?

219. Determine the unique pair of real numbers (x,y) that satisfy the equation
$(4x^2+6x+4)(4y^2-12y+25)=28$

220. Assume that x,y, and z are positive real numbers that satisfy the equations given
x + y+xy=8
y+z+yz=15
z +x+zx=35.
Determine the value of x +y+z+xyz.

221. The figure below shows the ellipse $\frac{(x-19)^2}{19}+\frac{(y-98)^2}{98}$=1998
Let R1, R2, R3, and R4 denote those areas within the ellipse that are in the 1st, 2nd, 3rd, and 4th quadrants, respectively. Determine the value of R1 − R2 + R3 − R.



222. Determine the three leftmost digits of the number
$1^1+2^2+3^3+..+1000^{100}$.

223. Find the positive integer whose square is exactly equal to the number
$\sum_{i=1}^{2001} (4\,i-2)^3+1$.

224. Factor $30\left(a^2+b^2+c^2+d^2\right)+68\,a\,b-75\,a\,c-156\,a\,d-61\,b\,c-100\,b\,d+87\,c\,d$.

225. It was recently shown that $2^{2^{24}}+1$ is not a prime number.
Find the four rightmost digits of this number.

226. Determine the value of

$$S=\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+...+\sqrt{1+\frac{1}{1999^2}+\frac{1}{2000^2}}$$

227. Find five different sets of three positive integers {k, m, n},
such that k < m < n and
$\frac{1}{k}+\frac{1}{m}+\frac{1}{n}=\frac{19}{84}$

228. Suppose $\frac{\cos 3x}{\cos x}=\frac{1}{3}$ for some angle x, $0\le x\le \pi/2$. Determine $\frac{\sin 3x}{\sin x}$ for the same x.

229. Let $p(x)=x^5+x^2+1$ have roots r1, r2, r3, r4, r5. Let $q(x)=x^2-2$. Determine the product
q(r1)q(r2)q(r3)q(r4)q(r5).

230. Assume that the irreducible fractions between 0 and 1, with denominators at most 99, are listed in ascending order. Determine which two fractions are adjacent to $\frac{17}{76}$in this listing.

231. Determine the smallest five-digit positive integer N such that 2N is also a five-digit integer and all ten digits from 0 to 9 are found in N and 2N.

232. Complete the square in the figure below with integers between 1 and 9 such that the sum of the numbers in each row, column is as indicated. The sum of elements in the right diagonal is 16 and the left diagonal is 20.

0	0	2	0	s14
0	5	0	0	s16
0	0	0	8	s26
3	0	0	0	s30
s21	s25	s13	s27	

233. Solve the following equations for real x and
$(3\,x+y)\,(x+3\,y)\,\sqrt{x\,y}\ =14$
$(x+y)\left(x^2+14\,x\,y+y^2\right)=36$

234. Find min value of max(a^2+b,b^2+a) for all real a and b.

235. Simplify $\sqrt[3]{20+14\sqrt{2}}+\sqrt[3]{20-14\sqrt{2}}$

236. Find the maximum value of $\sqrt[4]{r}-\frac{1}{\sqrt[4]{r}}$, given that $\sqrt[6]{r}+\frac{1}{\sqrt[6]{r}}=6$

237. Prove that $\left\|\begin{pmatrix}(x^2+1)^2 & (x\,y+1)^2 & (x\,z+1)^2\\ (x\,y+1)^2 & (y^2+1)^2 & (y\,z+1)^2\\ (x\,z+1)^2 & (y\,z+1)^2 & (z^2+1)^2\end{pmatrix}\right\|$= 2(y-z)^2(z-x)^2(x-y)^2

238. Is the polynomial P (x) = $x^{101}+101x^{100}+102$ irreducible over Z[x]?

239. The zeros of the polynomial P (x) = x^3 − 10x + 11 are u, v, and w. Determine the value of arctan u + arctan v + arctan w.

240. Find all triples(x, y, z) of real numbers that are solutions to the system of equations

$\frac{4\,x^2}{4\,x^2+1}=y$

$\frac{4\,y^2}{4\,y^2+1}=z$

$\frac{4\,z^2}{4\,z^2+1}=x$

241. Find the general term of the sequence given by $x_0=3$, $x_1=4$, and (n + 1)(n + 2)x_n = 4(n + 1)(n + 3)x_{n-1} − 4(n + 2)(n + 3)x_{n-2}, n ≥ 2.

242. Evaluate $\lim_{n\to\infty} n^2 \int_0^{\frac{1}{n}} x^{(x+1)}\,dx$

243. Simplify $\sum_{k=0}^{n}(-1)^k\,(n-k)!\,(n+k)!$

244. Find the minimum value of the real valued function $\frac{(x^2-x+1)^3}{x^6-x^3+1}$.

245. Integrate $\int\frac{x+\sin(x)-\cos(x)-1}{x+e^x+\sin(x)}\,dx$

246. Integrate $\int\frac{x^4+1}{x^6+1}\,dx$

247. Integrate $\int\sqrt{\frac{e^x+1}{e^x-1}}\,dx$

248. Integrate$\int\frac{x^2+1}{x^4-x^2+1}\,dx$

249. Integrate $\int(1+2\,x^2)\,e^{x^2}\,dx$

250. Find the global minimum of the function f(x,y) = $x^4+6\,x^2\,y^2+y^4-\frac{9\,x}{4}-\frac{7\,y}{4}$

251. Simplify sin(70 °) cos(50 °)+sin(260 °) cos(280 °)

252. Find the roots of the equation $x^3-3\,x=\sqrt{x+2}$

253. Let S be the set of all points of a unit cube (i.e., a cube each of whose edges is of length 1) that are at least as far from any of the vertices of the cube as from the center of the cube. Determine the shape and volume of S.

254. Solve the equation $2\sqrt[3]{2\,y-1}=y^3+1$ for real y.

255. Find the number of digits in 125^{100}.

256. Solve the equation y$(x+y)^2$=9 and y$(x^3$- $y^3)$ = 7 for real x and y.

257. Solve the equation x^4-14x^3+66x^2-115x+66.25=0

258. Calculate the sum of all divisors of the form $2^x\, 3^y$ (with x,y>0) of the number N = 19^{88}-1.

259. Solve the equations
$x^2 + y^2 + \frac{2\,x\,y}{x+y} = 1$
$\sqrt{x+y} = x^2 - y$

260. Solve the equations
$x^2 + y^2 + z^2 = 2$
$x + y + z = 2$

261. Let the number of different divisors of the integer n be N(n); e.g. 24 has the divisors 1,2,3,4,6,8,12 and 24, so N(24)=8. Determine whether the sum N(1) + N(2) + ... + N(1989) is odd or even.

262. Simplify $\sum_{k=1}^{2\,n-1} \frac{(-1)^{k-1}}{\binom{2\,n}{k}}$

263. All the 2-digit numbers from 19 to 93 are written consecutively to form the number N = 1920212...919293. Find the largest power of 3 that divides N.

264. Determine the largest 3 digit prime factor of the integer $\binom{2000}{1000}$.

265. Solve $(xy - 7)^2$= $x^2 + y^2$ for non-negative x and y.

266. Find the remainder when 2^{1990} is divided by 1990.

267. Consider the collection of all three element subsets drawn from the set{1,2,3,...,299,300}.Determine the number of subsets for which the sum of the three elements is a multiple of 3.

268. How many 3-element subsets of the set {1,2,3,...,19,20} are there such that the product of the three numbers in the subset is divisble by 4.

269. Let A denote the set of all numbers between 1 and 700 which are divisible by 3 and let B denote the set of all numbers between 1 and 300 which are divisible by 7. Find the number of all ordered pairs (a,b) such that a ∈ A,b ∈ B, a ≠b and a+b is even.

270. Find the greatest common divisor of all even 6 digit numbers obtained by using each of the digits 1,2,3,4,5,6 exactly once.

271. Find real roots of the equation $(x+1)\left(x^2+1\right)\left(x^3+1\right) = 30\,x^3$

272. Evaluate
$\sum_{k=1}^{\infty} \frac{1}{(2\,k-1)\,(2\,k+1)}$
$\sum_{k=1}^{\infty} \frac{1}{k\,(k+1)\,(k+2)}$

273. Factorize $(x+y+z)^5 - x^5 - y^5 - z^5$

274. Solve the equations
x_1+x_2+x_3=0
x_2+x_3+x_4=0
x_{98}+x_{99}+x_{100}=0
x_{100}+x_1+x_2=0

275. Sketch the curve $(2\,x+y)^2$(y-x)=x+y

276. Determine A_{1990} if A_{n+1}= $\frac{A_n}{1+\mathrm{n}A_n}$, n = 1,2,.. and A_0=A.

277. In setting the tpe for the multiplication (abc)(bca)(cab) = 234235286, with a > b > c, the setter pied all the digits except the units digits 6. Restore them to their proper order.

278. Find the maximum value of

279. Find the number of five digit numbers which are divisible by three and have 6 as their last digit.

280. Evaluate $\sum_{k=1}^{n} \frac{k^2}{2^k}$

281. Integrate $\int_0^\infty \frac{\sin^2(x)}{x^2}\, dx$

282. Evaluate the determinant

$$\left|\begin{pmatrix} 1 & a & a^2 & a^4 \\ 1 & b & b^2 & b^4 \\ 1 & c & c^2 & c^4 \\ 1 & d & d^2 & d^4 \end{pmatrix}\right|$$

283. Simplify $4 \tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}\left(\frac{1}{239}\right)$

284. Evaluate $\sum_{k=1}^{n} \cos^4\left(\frac{k\pi}{2n+1}\right)$

285. Find all real values of x,y and z satisfying the equations

$$x^2 + y^2 + z^2 = 9$$
$$x^4 + y^4 + z^4 = 33$$
$$x y z = -4$$

286. Find the value of $\alpha+\beta$ satisfying the equations

$$\alpha^3 - 3\alpha^2 + 5\alpha - 17 = 0$$
$$\beta^3 - 3\beta^2 + 5\beta + 11 = 0$$

287. Prove that $\prod_{k=1}^{n} 2^{\frac{k}{2^k}} < 4$

288. Find all integral solutions of the equation
1+1996 x+1998 y=x y

Solutions

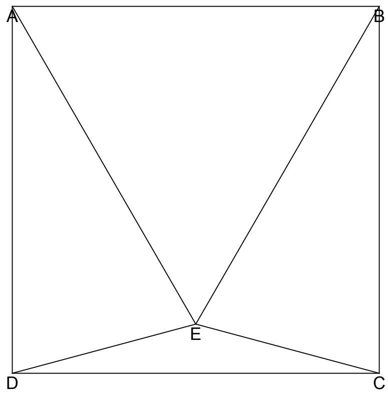
1. Solution

```
Module[{A = Point2D[{0, s}], B = Point2D[{s, s}],
   C = Point2D[{s, 0}], D = Point2D[{0, 0}],
   E = Point2D[{s / 2, s / 2 * Tan[15 Degree]}]},
  Print["E is ", Coordinates2D[E]];
  Print["AE = ", Simplify[Distance2D[A, E], s > 0]];
  Print["BE = ", Simplify[Distance2D[B, E], s > 0]];
  Block[{s = 1}, Show[Graphics[
     Line[{{A, B, C, D, A}, {A, E, D}, {B, E, C}} /. Point2D[{x_, y_}] → {x, y}]],
    Graphics[
     Label2D[#, CenteredBelow] & /@
      {{"A", A}, {"B", B}, {"C", C}, {"D", D}, {"E", E}}]
   ]
  ]
]
```

E is $\left\{\frac{s}{2}, \frac{1}{2}\left(2-\sqrt{3}\right) s\right\}$

AE = s

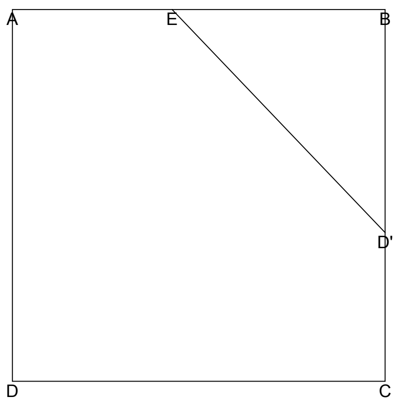
BE = s



2. Solution

```
Module[{D = Point2D[{0, 0}], D1 = Point2D[{s, v}],
   A = Point2D[{0, s}], B = Point2D[{s, s}], A1, E, C = Point2D[{s, 0}]},
  A1 = Reflect2D[A, Line2D[D, D1, Perpendicular2D]];
  E = Point2D[Line2D[A, B], Line2D[A1, D1]];
  Print["D'E + BE + D'B = ",
   Simplify[Distance2D[D1, E] + Distance2D[E, B] + Distance2D[D1, B],
    v > 0 && s > 0 && s > v]];
  Block[{s = 1, v = 0.4},
   Show[
    Graphics[Line[{{A, B, C, D, A}, {D1, E}} /. Point2D[{x_, y_}] → {x, y}]],
    Graphics[
     Label2D[#, CenteredBelow] & /@
      {{"A", A}, {"B", B}, {"C", C}, {"D", D}, {"E", E}, {"D'", D1}}]
   ]
  ]
]
```

```
D'E + BE + D'B = 2 s
```



3. Solution

```
Module[{O = Point2D[{0, 0}], A = Point2D[{-1, 0}], B = Point2D[{1, 0}],
   C = Point2D[{0, 1}], P = Point2D[{u, Sqrt[1 - u^2]}], BQ, QR, Q, R},
  Q = Point2D[Line2D[A, B], Line2D[C, P]];
  R = Point2D[Line2D[A, P], Line2D[Q, Line2D[A, B], Perpendicular2D]];
  BQ = Distance2D[B, Q];
  QR = Distance2D[Q, R];
  Print["Q is ", FullSimplify[Coordinates2D[Q], u > 0 && u < 1]];
  Print["R is ", FullSimplify[Coordinates2D[R], u > 0 && u < 1]];
  Print["BQ = ", FullSimplify[BQ, u > 0 && u < 1]];
  Print["QR = ", FullSimplify[QR, u > 0 && u < 1]];
  Print["BQ - QR = ", FullSimplify[BQ - QR, u > 0  && u < 1]];
  Block[{u = 0.8},
   Show[Graphics[
     Line[{{A, R}, {C, Q}, {A, Q}, {Q, R}} /. Point2D[{x_, y_}] → {x, y}]],
    Graphics[Circle[]],
    Graphics[
     Label2D[#, CenteredBelow] & /@
      {{"A", A}, {"B", B}, {"C", C}, {"O", O}, {"P", P}, {"Q", Q}, {"R", R}}]
   ]
  ]
]
```

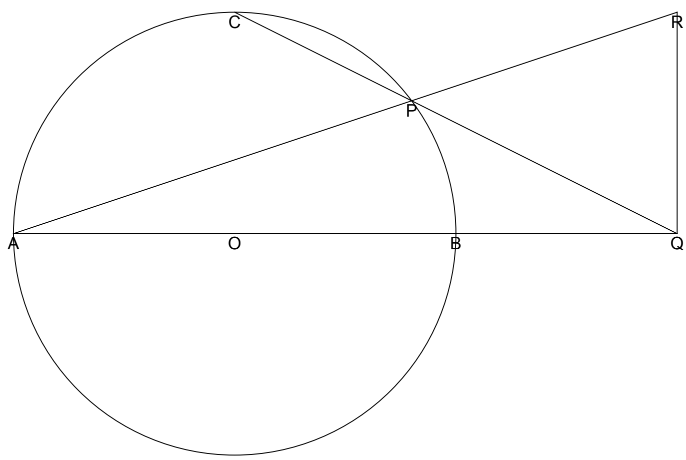
Q is $\left\{\frac{1+\sqrt{1-u^2}}{u}, 0\right\}$

R is $\left\{\frac{1+\sqrt{1-u^2}}{u}, \frac{1-u+\sqrt{1-u^2}}{u}\right\}$

BQ = $\frac{1-u+\sqrt{1-u^2}}{u}$

QR = $\sqrt{2}\sqrt{\frac{-1+u}{-1+\sqrt{1-u^2}}}$

BQ - QR = 0



4. Solution

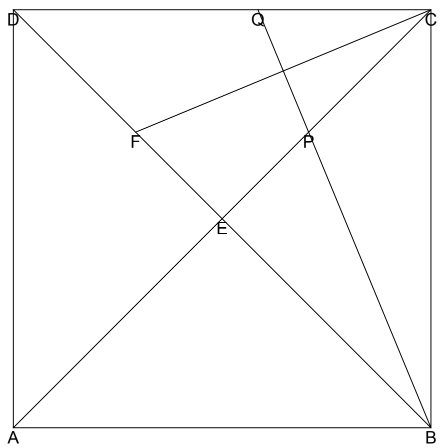
```
Module[{A = Point2D[{0, 0}], B = Point2D[{s, 0}],
   C = Point2D[{s, s}], D = Point2D[{0, s}],
   E = Point2D[{s / 2, s / 2 }], lCF, lBQ, P, Q, F},
  lCF = Line2D[C, Tan[45 / 2 Degree]];
  lBQ = Line2D[B, lCF, Perpendicular2D];
  P = Point2D[Line2D[A, C], lBQ];
  Q = Point2D[Line2D[C, D], lBQ];
  F = Point2D[Line2D[B, D], lCF];
  Print["P is ", FullSimplify[Coordinates2D[P], s > 0]];
  Print["Q is ", FullSimplify[Coordinates2D[Q], s > 0]];
  Print["PE = ", FullSimplify[Distance2D[P, E], s > 0]];
  Print["DQ = ", FullSimplify[Distance2D[D, Q], s > 0]];
  Block[{s = 1},
   Show[
    Graphics[Line[{{A, B, C, D, A}, {A, C}, {B, D}, {B, Q}, {C, F}} /.
       Point2D[{x_, y_}] → {x, y}]],
    Graphics[
     Label2D[#, CenteredBelow] & /@ {{"A", A}, {"B", B},
       {"C", C}, {"D", D}, {"E", E}, {"Q", Q}, {"P", P}, {"F", F}}]
   ]
  ]
]
```

P is $\left\{\frac{s}{1+\text{Tan}\left[\frac{\pi}{8}\right]}, \frac{s}{1+\text{Tan}\left[\frac{\pi}{8}\right]}\right\}$

Q is $\left\{s - s\,\text{Tan}\left[\frac{\pi}{8}\right], s\right\}$

PE = $\frac{1}{2}\left(s - s\,\text{Tan}\left[\frac{\pi}{8}\right]\right)$

DQ = $s - s\,\text{Tan}\left[\frac{\pi}{8}\right]$



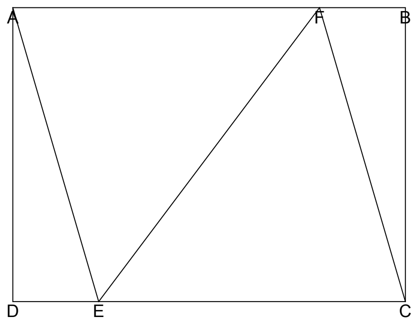
5. Solution

```
Module[{A = Point2D[{0, 12}], B = Point2D[{16, 12}],
   C = Point2D[{16, 0}], D = Point2D[{0, 0}],
   E, F, lEF},
  lEF = Line2D[Point2D[D, B], Line2D[A, C], Perpendicular2D];
  E = Point2D[lEF, Line2D[D, C]];
  F = Point2D[lEF, Line2D[A, B]];
  Print["E is ", Coordinates2D[E]];
  Print["F is ", Coordinates2D[F]];
  Print["EF = ", Simplify[Distance2D[E, F]]];
  Show[
   Graphics[
    Line[{{A, B, C, D, A}, {A, E}, {E, F}, {C, F}} /. Point2D[{x_, y_}] → {x, y}]],
   Graphics[
    Label2D[#, CenteredBelow] & /@
     {{"A", A}, {"B", B}, {"C", C}, {"D", D}, {"E", E}, {"F", F}}]
  ]
]
```

E is $\left\{\frac{7}{2}, 0\right\}$

F is $\left\{\frac{25}{2}, 12\right\}$

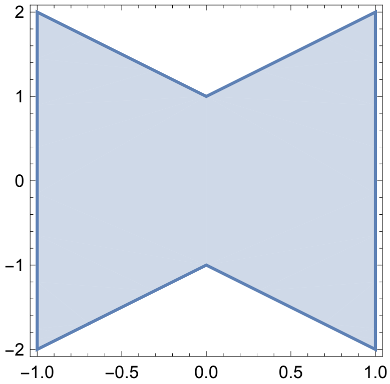
EF = 15



6. Solution

```
Module[{R},
 R = ImplicitRegion[Abs[x] - Abs[y] ≤ 1 && Abs[y]  ≤ 1, {{y , -4, 4}, {x, -4, 4}}];
 Print["The area of the region is ", RegionMeasure[R]];
 RegionPlot[R]
]
```

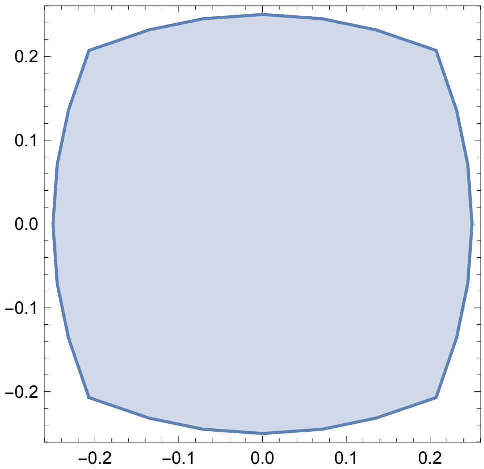
The area of the region is 6



7. Solution

```
Module[{R},
 R = ImplicitRegion[Sqrt[x^2 + y^2] ≤ Min[Abs[x + 1 / 2 ], Abs[1 / 2 - x ],
     Abs[1 / 2 - y], Abs[y + 1 / 2]], {{x , -1 / 2, 1 / 2}, {y, -1 / 2, 1 / 2}}];
 Print["The required probability is ", FullSimplify[RegionMeasure[R]]];
 RegionPlot[R]
]
```

The required probability is $\frac{1}{3}\left(-5+4\sqrt{2}\right)$



8. Solution

```
Module[{A, B, I},
 I = Integrate[Sqrt[1 - x^2], x];
 A = (I /. x → b) - (I /. x → a);
 B = (I /. x → Sqrt[1 - a^2]) - (I /. x → Sqrt[1 - b^2]);
 Print["A + B = ", FullSimplify[A + B, a > 0 && b > 0 && a < b]];
 Print["Length of the arc is ",
  FullSimplify[
   ArcSin[Sqrt[1 - a^2]] - ArcSin[Sqrt[1 - b^2]],
   a > 0 && b > 0 && a < b]];
]
```

```
A + B = -ArcSin[a] + ArcSin[b]

Length of the arc is ArcCos[a] - ArcCos[b]
```

9. Solution

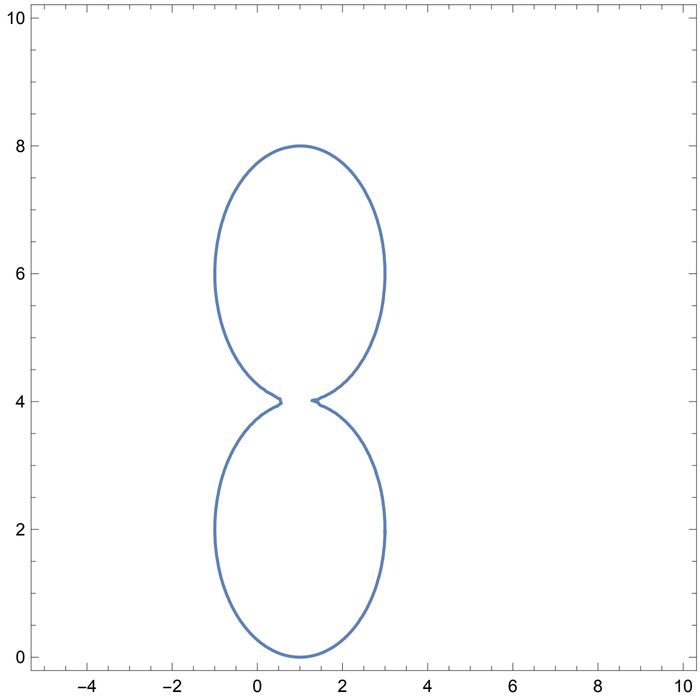
```
Reduce[
 2 * r * Sin[a / 2] == 1 &&
  2 * r * Sin[b / 2] == 2 &&
  2 * r * Sin[c / 2] == 3 &&
  a + b + c == Pi && a > 0 && b > 0 && c > 0 , {a, b, c}]
```

```
r == Root[-3 - 7 #1 + 2 #1^3 &, 3] &&
 a == 4 ArcTan[Root[{-3 - 7 #1 + 2 #1^3 &, 1 - 4 #1 #2 + #2^2 &}, {3, 1}]] &&
 b == -4 ArcTan[Root[{-3 - 7 #1 + 2 #1^3 &, 1 - 4 #1 #2 + #2^2 &,
      -9 - 9 #1 + 7 #1^2 - 6 #2 - #1 #2 + 3 #1^2 #2 + 5 #1 #3 &}, {3, 1, 1}]] && c == -a - b + π
```

10. Solution

```
Module[{A = Point2D[{a, b}], Q = Point2D[{x, y}], f, P},
 P = Point2D[A, Q, 1, m];
 f = FullSimplify[
   Eliminate[{Distance2D[A, P] * Distance2D[A, Q] - k^2 == 0,
     YCoordinate2D[P] == 0}, m], a > 0 && b > 0 && k > 0];
 Print["The locus of Q is ", f];
 Block[{a = 1, b = 4, k = 4},
  Print[ContourPlot[Evaluate[f], {x, -5, 10}, {y, 0, 10}]];
  Print[f]];
]
```

The locus of Q is $b^2 \left((a-x)^2+(b-y)^2\right)^2 = k^4 (b-y)^2$

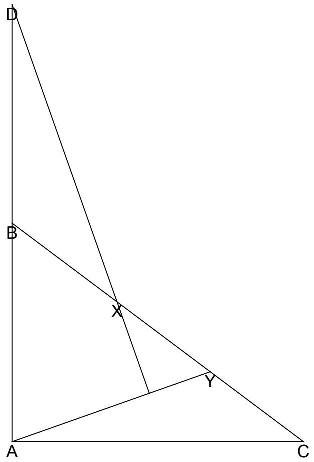


$16 \left((1-x)^2+(4-y)^2\right)^2 = 256 (4-y)^2$

11. Solution

```
Module[{A = Point2D[{0, 0}], B = Point2D[{0, s}],
  C = Point2D[{t, 0}], D = Point2D[{0, 2 s}], X, Y, Z},
X = Point2D[Line2D[B, C], Line2D[A, Line2D[B, C], Perpendicular2D]];
 Y = Point2D[X, C]; Z = Point2D[Line2D[D, X], Line2D[A, Y]];
Print["Product of slopes of DX and AY =",
  FullSimplify[Slope2D[Line2D[D, X]] * Slope2D[Line2D[A, Y]], s > 0 && t > 0]];
 Block[{s = 3, t = 4},
  Show[
   Graphics[
    Line[{{A, B, C, A}, {B, D}, {D, Z}, {A, Y}} /. Point2D[{x_, y_}] → {x, y}]],
   Graphics[
    Label2D[#, CenteredBelow] & /@
     {{"A", A}, {"B", B}, {"C", C}, {"D", D}, {"X", X}, {"Y", Y}}]
  ]
 ]
]
```

```
Product of slopes of DX and AY =-1
```



12. Solution

```
Module[{C = Point2D[{c, 0}], B = Point2D[{0, 0}], lAB, lBD, A, D, Da},
 lAB = Line2D[B, Tan[120 Degree]];
 A = Point2D[B, lAB, -1];
 lBD = Line2D[B, lAB, Perpendicular2D];
 D = Point2D[C, Line2D[C, A], -1];
 Da = Point2D[lBD, Line2D[C, A]];
 Solve[XCoordinate2D[D] == XCoordinate2D[Da], c]]
```

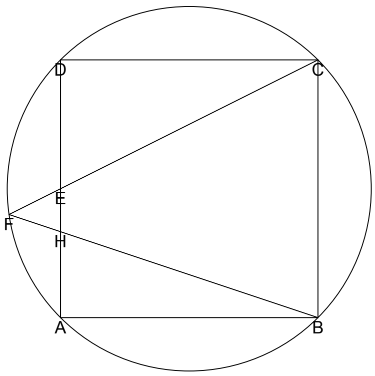
$\left\{\left\{c \to -\frac{1}{2}\right\}, \left\{c \to (-2)^{2/3}\right\}, \left\{c \to 2^{2/3}\right\}, \left\{c \to -(-1)^{1/3}\, 2^{2/3}\right\}\right\}$

13. Solution

```
Module[{A = Point2D[{-s, -s}], B = Point2D[{s, -s}],
   C = Point2D[{s, s}], D = Point2D[{-s, s}], E, F, H},
E = Point2D[A, D];
F = Points2D[Line2D[C, E], Circle2D[A, B, C]][[2]];
 H = Point2D[Line2D[F, B], Line2D[A, D]];
 Print["HD =", Simplify[Distance2D[H, D], s > 0]];
 Print["AH =", Simplify[Distance2D[A, H], s > 0]];
 Block[{s = 3},
  Show[
   Graphics[Circle[{0, 0}, s √2]],
   Graphics[Line[{{A, B, C, D, A}, {F, B}, {C, F}, {H, D}, {A, H}} /.
      Point2D[{x_, y_}] → {x, y}]],
   Graphics[
    Label2D[#, CenteredBelow] & /@
     {{"A", A}, {"B", B}, {"C", C}, {"D", D}, {"E", E}, {"F", F}, {"H", H}}]
  ]
 ]
]
```

HD $= \frac{4\,s}{3}$

AH $= \frac{2\,s}{3}$



14. Solution

```
Module[{A = Point2D[{0, 0}], B = Point2D[{l, 0}], C = Point2D[{l, h}],
   D = Point2D[{0, h}], E = Point2D[{e, 0}], F = Point2D[{0, f}]},
  FullSimplify[Eliminate[{
     Area2D[Triangle2D[A, E, F]] == 12,
     Area2D[Triangle2D[E, B, C]] == 16,
     Area2D[Triangle2D[F, C, D]] == 30}, {e, f}]]
]
```

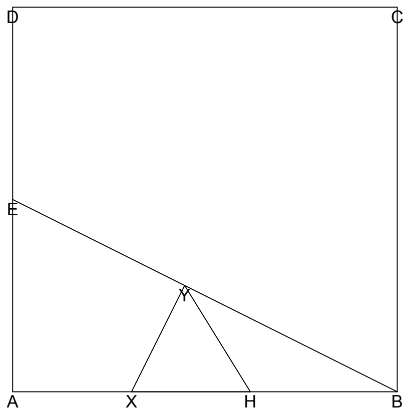
(-96 + h l) (-20 + h l) == 0

15. Solution

```
Module[{A = Point2D[{0, 0}], B = Point2D[{s, 0}], C = Point2D[{s, s}],
   D = Point2D[{0, s}], E, X, Y, H = Point2D[{h, 0}], sol},
  E = Point2D[A, D];
  X = Point2D[A, H];
  Y = Point2D[ Line2D[E, B], Line2D[X, Line2D[E, B], Perpendicular2D]];
  sol = Solve[
     Distance2D[A, B] * Distance2D[B, H] == Distance2D[A, H]^2, h, Reals][[2]];
  H = H /. sol; Y = Y /. sol; X = X /. sol;
  Print["XY =", Simplify[Distance2D[X, Y] , s > 0]];
  Print["YH =", Simplify[Distance2D[X, H] , s > 0]];
  Block[{s = 4},
   Show[
    Graphics[Line[{{A, B, C, D, A}, {E, B}, {X, Y}, {X, H}, {Y, H}} /.
        Point2D[{x_, y_}] → {x, y}]],
    Graphics[
     Label2D[#, CenteredBelow] & /@ {{"A", A}, {"B", B},
        {"C", C}, {"D", D}, {"E", E}, {"X", X}, {"Y", Y}, {"H", H}}]
    ]
   ]
]
```

$$XY = \frac{1}{2}\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}\ s$$

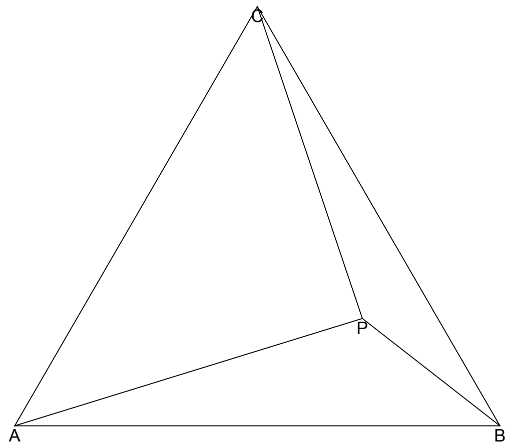
$$YH = \frac{1}{2}\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}\ s$$



16. Solution

```
Module[{A = Point2D[{0, 0}], B = Point2D[{s, 0}],
  C = Point2D[{s / 2, s √3 / 2}], P = Point2D[{x, y}]},
sol = Solve[Distance2D[A, P] == 3 && Distance2D[B, P] == 4 &&
     Distance2D[C, P] == 5, {x, y, s}][[3]];
 P = P /. sol;
 Print["Area of ABC = ", FullSimplify[√3 s^2 / 4 /. sol]];
 Block[{s = 4},
  Show[
   Graphics[
    Line[{{A, B, C, A}, {P, A}, {P, B}, {P , C}} /. Point2D[{x_, y_}] → {x, y}]],
   Graphics[
    Label2D[#, CenteredBelow] & /@ {{"A", A}, {"B", B}, {"C", C}, {"P", P }}]
  ]
 ]
]
```

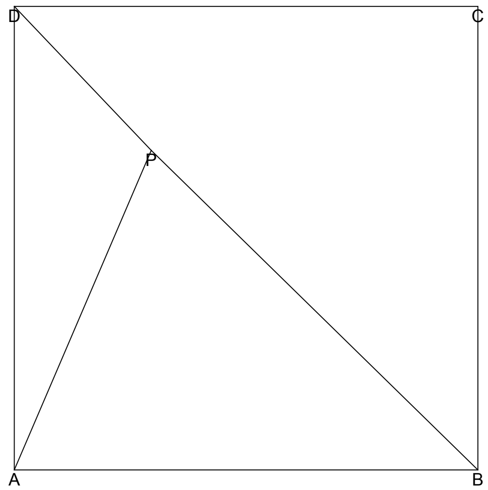
Area of ABC = $9 + \frac{25\sqrt{3}}{4}$



17. Solution

```
Module[{A = Point2D[{0, 0}], B = Point2D[{s, 0}],
   C = Point2D[{s, s}], D = Point2D[{0, s}], P = Point2D[{x, y}]},
sol = Solve[Distance2D[A, P] == 3 && Distance2D[B, P] == 7 &&
      Distance2D[D, P] == 5, {x, y, s}][[4]];
 Print["Area of ABCD = ", FullSimplify[s^2 /. sol]];
 P = P /. sol;
 Block[{s = 4},
  Show[
   Graphics[Line[
     {{A, B, C, D, A}, {P, A}, {P, B}, {P , D}} /. Point2D[{x_, y_}] → {x, y}]],
   Graphics[
    Label2D[#, CenteredBelow] & /@
     {{"A", A}, {"B", B}, {"C", C}, {"D", D}, {"P", P }}]
  ]
 ]
]
```

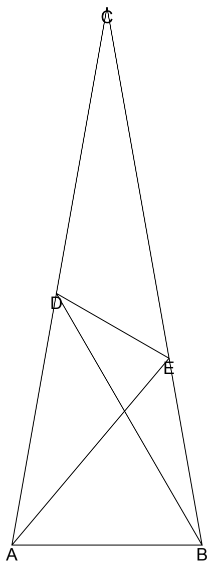
```
Area of ABCD = 58
```



18. Solution

```
Module[{A = Point2D[{0, 0}], B, C, D, E},
 B = Point2D[{2 * l * Sin[10 Degree], 0}];
 C = Point2D[{l * Sin[10 Degree], l * Cos[10 Degree]}];
 D = Point2D[Line2D[A, C], Line2D[B, Tan[120 Degree]]];
 E = Point2D[Line2D[B, C], Line2D[A, Tan[50 Degree]]];
 Print["∡EDB = ", N[Angle2D[Line2D[D, B], Line2D[D, E]] / Degree, 3]];
Block[{l = 10},
  Show[
   Graphics[Line[{{A, B, C, A, E, D, B}} /. Point2D[{x_, y_}] → {x, y}]],
   Graphics[
    Label2D[#, CenteredBelow] & /@
     {{"A", A}, {"B", B}, {"C", C}, {"D", D}, {"E", E }}]
  ]
 ]
]
```

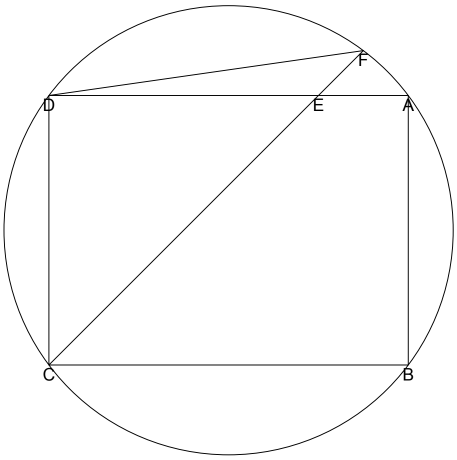
∡EDB = 30.0



19. Solution

```
Module[{A = Point2D[{4, 3}], B = Point2D[{4, -3}],
   C = Point2D[{-4, -3}], D = Point2D[{-4, 3}], E = Point2D[{2, 3}], F},
 F = Points2D[Line2D[C, E], Circle2D[A, B, C]][[2]];
  Print["FD = ", Distance2D[F, D]];
  Show[
   Graphics[Circle[{0, 0}, 5]],
   Graphics[
    Line[{{A, B, C, D, A}, {F, D}, {C, F}} /. Point2D[{x_, y_}] → {x, y}]],
   Graphics[
    Label2D[#, CenteredBelow] & /@
     {{"A", A}, {"B", B}, {"C", C}, {"D", D}, {"E", E}, {"F", F}}]
  ]
 ]
```

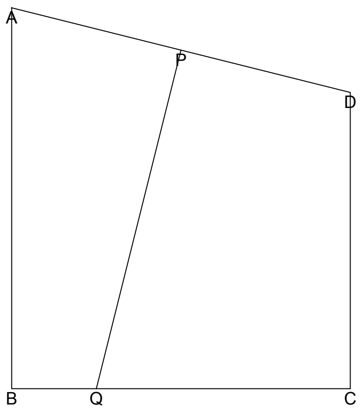
FD = $5\sqrt{2}$



20. Solution

```
Module[{A = Point2D[{0, 9}], B = Point2D[{0, 0}],
   C = Point2D[{8, 0}], D = Point2D[{8, 7}], P, Q},
P = Point2D[A, D];
 Q = Point2D[Line2D[B, C], Line2D[P, Line2D[A, D], Perpendicular2D]];
Print["Area of APBQ = ",
   Area2D[Triangle2D[A, B, Q]] + Area2D[Triangle2D[B, P, Q]]];
Show[
Graphics[Line[{{A, B, C, D, A}, {P, Q}} /. Point2D[{x_, y_}] → {x, y}]],
Graphics[
Label2D[#, CenteredBelow] & /@
     {{"A", A}, {"B", B}, {"C", C}, {"D", D}, {"P", P}, {"Q", Q}}]
]
]
```

```
Area of APBQ = 17
```



21. Solution

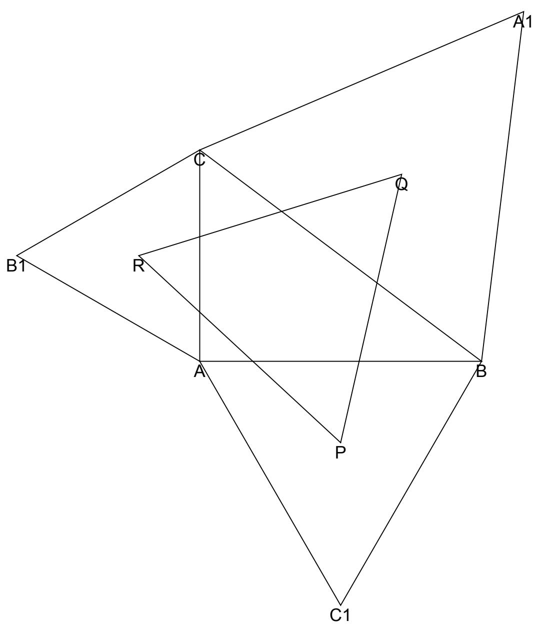
```
Module[{A, B, C, A1, B1, C1, P, Q, R, j},
 A = a;
 B = b;
 C = c;
 j = Exp[I * Pi / 3];
 A1 = C + (B - C) * j;
 B1 = A + (C - A) * j;
 C1 = B + (A - B) * j;
 P = (A + B + C1) / 3;
 Q = (B + C + A1) / 3;
 R = (C + A + B1) / 3;
 Print["P is ", P];
 Print["Q is ", Q];
 Print["R is ", R];
 Print["P -ω² (Q-P) - R = ", FullSimplify[P + j * (Q - P ) - R]];
 Block[{ a = 0, b = 4, c = 3 * I},
  Show[Graphics[
    Line[{ReIm /@ {A, B, C, A, C1, B, A1, C, B1, A}, ReIm /@ {P, Q, R, P}} ]],
   Graphics[Text[#[[1]], ReIm[#[[2]]], {0, 1}] & /@ {{"A", A}, {"B", B},
      {"C", C}, {"C1", C1}, {"A1", A1}, {"B1", B1}, {"P", P}, {"Q", Q}, {"R", R}}]
  ]
 ]
]
```

P is $\frac{1}{3}\left(a+2\,b+(a-b)\,e^{\frac{i\pi}{3}}\right)$

Q is $\frac{1}{3}\left(b+2\,c+(b-c)\,e^{\frac{i\pi}{3}}\right)$

R is $\frac{1}{3}\left(2\,a+c+(-a+c)\,e^{\frac{i\pi}{3}}\right)$

P $-\omega^2$ (Q−P) − R = 0



22. Solution

```
Module[{A0, A1, A2},
 A0 = 1;
 A1 = Exp[I * 2 * Pi / 5];
 A2 = Exp[I * 4 * Pi / 5];
 Print["(A0A1 * A0A2)^2 = ", FullSimplify[(Abs[A0 - A1] * Abs[A0 - A2])^2]];
]
```

$(A_0A_1 * A_0A_2)^2 = 5$

23. Solution

```
Module[{A0, A1, A2, A3},
 A0 = 1;
 A1 = Exp[I * 2 * Pi / 7];
 A2 = Exp[I * 4 * Pi / 7];
 A3 = Exp[I * 6 * Pi / 7];
 Print["1/(A0 A1) - 1/(A0 A2) - 1/(A0 A3) = ",
  FullSimplify[1 / Abs[A0 - A1] - 1 / Abs[A0 - A2] - 1 / Abs[A0 - A3]]];
]
```

$\frac{1}{A_0 A_1} - \frac{1}{A_0 A_2} - \frac{1}{A_0 A_3} = 0$

24. Solution

```
Module[{A = 0, B = a + I * b, D = c + I * d, C},
 C = B + D;
 Print["AB^2+BC^2+CD^2+DA^2 = ", FullSimplify[
   (Abs[B - A])^2 + (Abs[B - C])^2 + (Abs[D - C])^2 + (Abs[D - A])^2,
   a > 0 && b > 0 && c > 0 && d > 0]];
  Print["AC^2+BD^2 = ", FullSimplify[
   (Abs[C - A])^2 + (Abs[B - D])^2, a > 0 && b > 0 && c > 0 && d > 0]];
]
```

$AB^2+BC^2+CD^2+DA^2 = 2\ (a^2 + b^2 + c^2 + d^2)$

$AC^2+BD^2 = 2\ (a^2 + b^2 + c^2 + d^2)$

25. Solution

```
Module[{A = 0, B = a, C = b + I * c, G},
 G = (A + B + C) / 3;
 Print["AB²+BC²+CA² = ", FullSimplify[
   (Abs[B - A]) ^2 + (Abs[B - C]) ^2 + (Abs[C - A]) ^2 , a > 0 && b > 0 && c > 0 ]];
  Print["3 (GA²+GB²+GC²) = ", FullSimplify[
   3 * ((Abs[G - A]) ^2 + (Abs[G - B]) ^2 + (Abs[G - C]) ^2), a > 0 && b > 0 && c > 0 ]];
]
```

AB²+BC²+CA² = 2 (a² − a b + b² + c²)

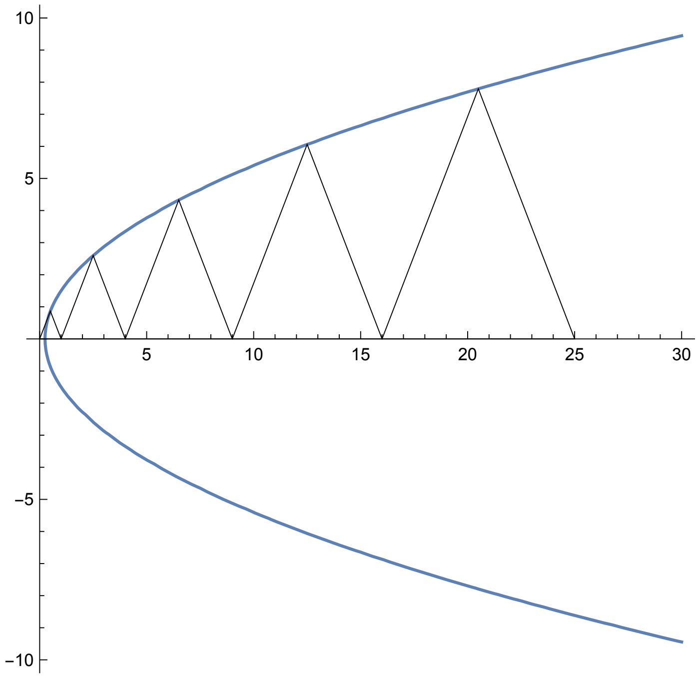
3 (GA²+GB²+GC²) = 2 (a² − a b + b² + c²)

26. Solution

```
Module[{t, lt, p},
 t = Triangle2D[{0, 0}, {1 / 2, √3 / 2}, {1, 0}];
 lt = Table[ Translate2D[Scale2D[t, 2 * i - 1], {(i - 1) ^2, 0}], {i, 1, 5}];
 p = List @@ FullSimplify[Apply[Quadratic2D, Point2D[#, 2] & /@ lt]].
    {x^2, xy, y^2, x, y, 1} == 0;
 Print["Distances of the vertices from the focus (1,0) = " ,
  Distance2D[Point2D[{1, 0}], Point2D[#, 2]] & /@ lt];
 Print["The equation of the parabola is ", p];
 Show[ContourPlot[Evaluate[p], {x, 0, 30}, {y, -10, 10}],
  Sketch2D[lt], Frame → False, Axes → True ]
]
```

Distances of the vertices from the focus (1,0) = {1, 3, 7, 13, 21}

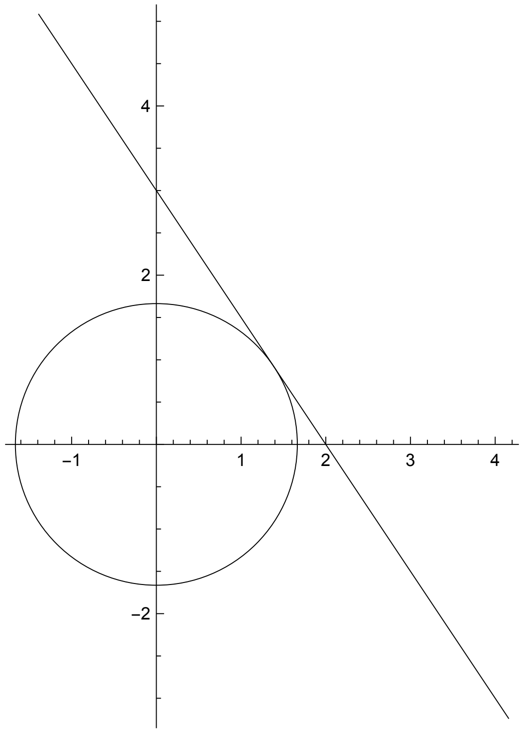
The equation of the parabola is $3 - 12 x + 4 y^2 == 0$



27. Solution

```
Module[{sol},
 sol = Solve[Distance2D[Point2D[{0, 0}], Line2D[b, a, -a * b]] == c, c][[1]];
 Print[sol];
 Block[{a = 2, b = 3},
  Show[
   Graphics[Circle[{0, 0}, c /. sol]],
   Sketch2D[{Line2D[b, a, -a * b]}], Axes → True
  ]
 ]
]
```

$$\left\{c \to \sqrt{\frac{a^2 b^2}{a^2 + b^2}}\right\}$$



28. Solution

```
lc = CoefficientList[Eliminate[{(x - h)^2 + (y - k)^2 == r^2 , y^2 == a * x}, x] /.
    {lhs_ == rhs_ → lhs - rhs}, y ];
First[
  lc] /
 Last[
  lc]
```

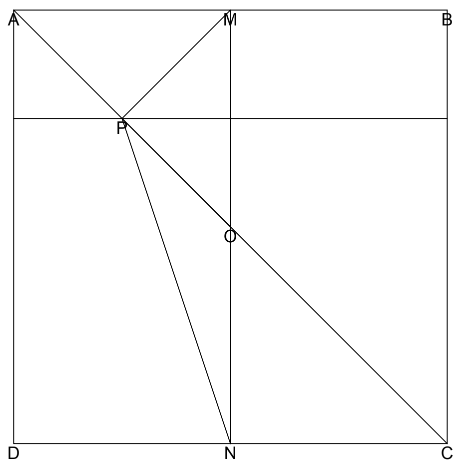
$a^2 h^2 + a^2 k^2 - a^2 r^2$

29. Solution

```
Module[{O = Point2D[{0, 0}], A = Point2D[{-s, s}],
   B = Point2D[{s, s}], C = Point2D[{s, -s}], D = Point2D[{-s, -s}],
   P, M = Point2D[{0, s}], N = Point2D[{0, -s}], Q = Point2D[{0, u}]},
  P = Point2D[Line2D[A, C], Line2D[Q, Line2D[A, B], Parallel2D]];
  Print["OP^4 + (MN/2)^4 = ", Simplify[Distance2D[O, P]^4 + s^4 , s > 0]];
  Print["MP^2.NP^2 = ", Simplify[Distance2D[M, P]^2 * Distance2D[N, P]^2 , s > 0]];
  Block[{s = 4, u = 2},
   Show[
    Graphics[Line[{{A, B, C, D, A}, {M, N}, {M, P}, {N, P}, {O, P}, {A, C},
        {Point2D[{-s, u}], Point2D[{s, u}]}} /. Point2D[{x_, y_}] → {x, y}]],
    Graphics[
     Label2D[#, CenteredBelow] & /@ {{"A", A}, {"B", B},
       {"C", C}, {"D", D}, {"M", M}, {"N", N}, {"P", P}, {"O", O}}]
   ]
  ]
]
```

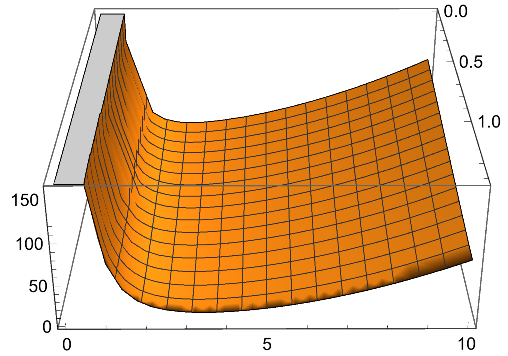
$OP^4 + \left(\frac{MN}{2}\right)^4 = s^4 + 4 u^4$

$MP^2.NP^2 = s^4 + 4 u^4$



30. Solution

```
Plot3D[(u - v)^2 + (Sqrt[2 - u^2] - 9 / v)^2, {u, 0, Sqrt[2]}, {v, 0, 10}]
Minimize[{(u - v)^2 + (Sqrt[2 - u^2] - 9 / v)^2, u > 0, u < Sqrt[2]}, {u, v}]
```

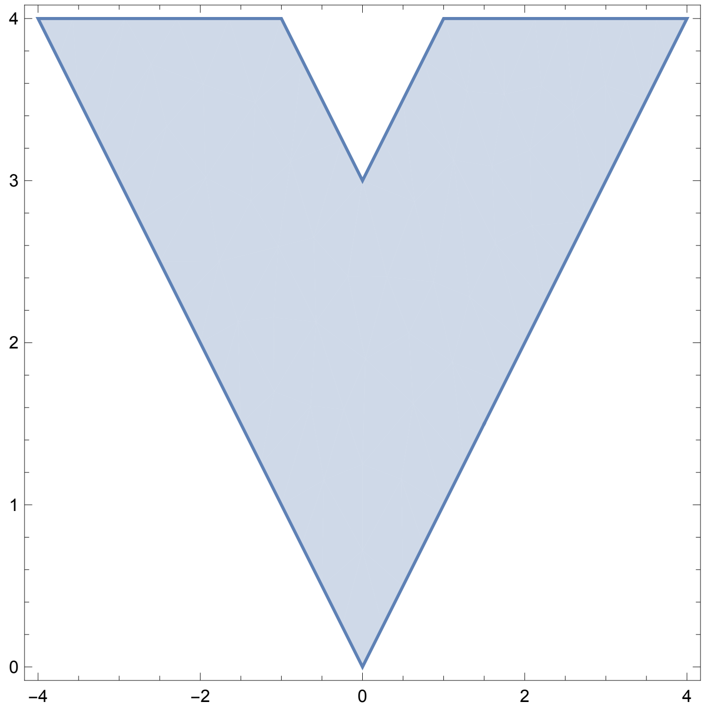



```
{8, {u → 1, v → 3}}
```

31. Solution

```
Module[{R},
 R =
  ImplicitRegion[Abs[x] ≤ y && y ≤ Abs[x ] + 3 && y ≤ 4, {{x , -10, 10}, {y, 0, 4}}];
 Print["The region centroid is ", FullSimplify[RegionCentroid[R]]];
 RegionPlot[R]
]
```

The region centroid is $\left\{0, \frac{13}{5}\right\}$



32. Solution

```
Minimize[{(r - 1)^2 + (s / r - 1)^2 + (t / s - 1)^2 + (4 / t - 1)^2,
   1 ≤ r, r ≤ s, s ≤ t, t ≤ 4}, {r, s, t}][[1]]
```

$$\left\{\left(-1+\sqrt{\frac{13}{2}+3\sqrt{2}}\right)^2+\left(-1+\frac{s}{\sqrt{\frac{13}{2}+3\sqrt{2}}}\right)^2+\left(-1+\frac{4}{t}\right)^2+\left(-1+\frac{t}{s}\right)^2,\right.$$

$$\left.\text{True},\ \sqrt{\frac{13}{2}+3\sqrt{2}}\le s,\ s\le t,\ t\le 4\right\}$$

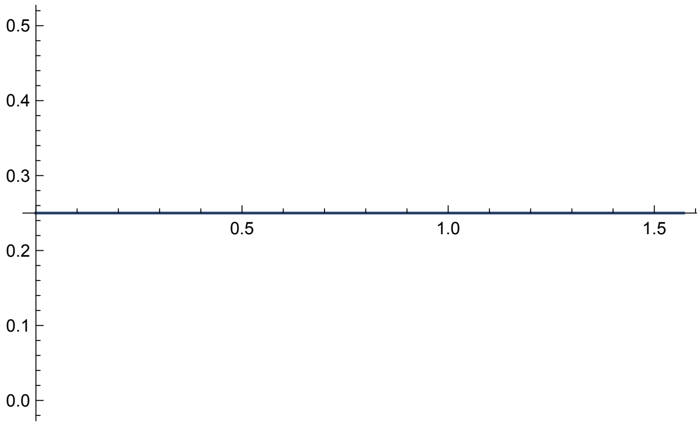
33. Solution

```
FullSimplify[Reduce[
    2 * r * Sin[a / 2] == 3 &&
     2 * r * Sin[b / 2] == 2 &&
     4 a + 4 b == 2 * Pi && a > 0 && b > 0 , {a, b}]];
r = Sqrt[13/2 + 3 Sqrt[2]] ;
Print["Area is ", FullSimplify[2 * Sqrt[r^2 - 9 / 4] * 3 + 2 * Sqrt[r^2 - 1] * 2]];
```

Area is $13+12\sqrt{2}$

34. Solution

```
Module[{R , Rt},
 R = ImplicitRegion[x ≥ -1 / 2 && x ≤ 1 / 2 && y ≤ 1 / 2 && y >= -1 / 2, {x, y}];
 Rt[x_] := TransformedRegion[TransformedRegion[R,
     TranslationTransform[{1 / 2, 1 / 2}]], RotationTransform[x ]];
 Plot[FullSimplify[RegionMeasure[RegionIntersection[R, Rt[x]]]], {x, 0, Pi/2}]
]
```



35. Solution

```
Sum[Sum[2^i, {i, 0, k - 1}], {k, 1, n }]
```

$-2+2^{1+n}-n$

36. Solution

```
Module[{f, f5, f5e},
 f[x_] := (2 * x - 1) / (x + 1);
 f5e = Simplify[Nest[f, x, 5]];
 f5[x_] := Evaluate[f5e];
 Print["f5=", f5e ];
 Print["f35=", Simplify[Nest[f5, x, 7]]];
 Print["f28=", Simplify[Nest[f, Nest[f5, x, 5], 3]]];
]
```

$f_{5=}\frac{1+x}{2-x}$

$f_{35=}\frac{1+x}{2-x}$

$f_{28=}\frac{-1+x}{x}$

37. Solution

```
Select[Range[60, 70], Divisible[(2^48) - 1, #] &]
```

{63, 65}

38. Solution

```
Mod[#^1982 + 1, # - 1] & /@ Range[3, 5]
```

{0, 2, 2}

39. Solution

```
RSolve[{u[n + 1] == u[n] + n^2 + 3 n + 3, u[0] == 5}, u[n], n] /. n → 1
```

{{u[1] → 8}}

40. Solution

```
Print["∑_{n=2}^{k} (n^2 - 2 n - 4)/(n^4 + 4 n^2 + 16)=",
 Sum[(n^2 - 2 n - 4) / (n^4 + 4 n^2 + 16), {n, 2, k}] // TraditionalForm]
Print["∑_{n=2}^{∞} (n^2 - 2 n - 4)/(n^4 + 4 n^2 + 16)= ",
 Limit[Sum[(n^2 - 2 n - 4) / (n^4 + 4 n^2 + 16), {n, 2, k}], k → Infinity]]
```

$$\sum_{n=2}^{k}\frac{n^2-2n-4}{n^4+4n^2+16}=\frac{k^4-12k^3-29k+40}{14\,(k^2+3)\,(k^2+2k+4)}$$

$$\sum_{n=2}^{\infty}\frac{n^2-2n-4}{n^4+4n^2+16}=\frac{1}{14}$$

41. Solution

```
Print["∫_0^2 (16 - x^2) x/(16 - x^2 + √((4 - x) (4 + x) (12 + x^2))) dx = ",
 Integrate[(16 - x^2) x / (16 - x^2 + Sqrt[(4 - x) (4 + x) (12 + x^2)]), {x, 0, 2}]]
```

$$\int_0^2\frac{(16-x^2)\,x}{16-x^2+\sqrt{(4-x)\,(4+x)\,(12+x^2)}}\,dx = 1$$

42. Solution

```
Module[{X, Y, M, A, B},
 X = {{7, 8, 9}, {8, -9, -7}, {-7, -7, 9}};
 Y = {{9, 8, -9}, {8, -7, 7}, {7, 9, 8}};
 A = Inverse[Y] - X;
 B = Inverse[Inverse[X] + Inverse[A]];
 Print["M = ", MatrixPower[X.Y - B.Y, 0.5] // TraditionalForm]]
```

$$M = \begin{pmatrix} 190. + 0.\,i & 81. + 0.\,i & 65. + 0.\,i \\ -49. + 0.\,i & 64. + 0.\,i & -191. + 0.\,i \\ -56. + 0.\,i & 74. + 0.\,i & 86. + 0.\,i \end{pmatrix}$$

43. Solution

```
Integrate[(Cos[x]^4 + Sin[x] Cos[x]^3 + Sin[x]^2 * Cos[x]^2 + Sin[x]^3 * Cos[x]) /
   (Sin[x]^4 + Cos[x]^4 + 2 Sin[x] Cos[x]^3 +
     2 Sin[x]^2 * Cos[x]^2 + 2 Sin[x]^3 * Cos[x]), x] // TraditionalForm
```

$$\frac{x}{2} + \frac{1}{2}\log(\sin(x) + \cos(x))$$

```
Integrate[(Cos[x]^4 + Sin[x] Cos[x]^3 + Sin[x]^2 * Cos[x]^2 + Sin[x]^3 * Cos[x]) /
  (Sin[x]^4 + Cos[x]^4 + 2 Sin[x] Cos[x]^3 +
    2 Sin[x]^2 * Cos[x]^2 + 2 Sin[x]^3 * Cos[x]), {x, 0, Pi / 2}]
```

$$\frac{\pi}{4}$$

44. Solution

```
Print["∑_{i=1}^{n} (i^2 - 2)/(i + 2)! =", Sum[(i^2 - 2) / Factorial[i + 2], {i, 1, n}]]
Print["∑_{i=1}^{∞} (i^2 - 2)/(i + 2)! =",
 Limit[Sum[(i^2 - 2) / Factorial[i + 2], {i, 1, n}], n → Infinity]]
```

$$\sum_{i=1}^{n} \frac{i^2 - 2}{(i + 2)\,!} = -\frac{n\,(3 + n)}{(3 + n)\,!}$$

$$\sum_{i=1}^{\infty} \frac{i^2 - 2}{(i + 2)\,!} = 0$$

45. Solution

```
Factorial[40]
```

815 915 283 247 897 734 345 611 269 596 115 894 272 000 000 000

46. Solution

```
Fold[Plus, 0, Length[Permutations[#]] & /@ IntegerPartitions[16, All, {1, 3}]]
```

277

47. Solution

```
Integrate[(x - 2) / ((x^2 + 4) * Sqrt[x]), {x, 1, 4}] // Simplify
```

0

48. Solution

```
Integrate[1 / (2 + Tan[θ]), {θ, 0, Pi / 4}]
```

$\frac{1}{10}\left(\pi + \text{Log}\left[\frac{9}{8}\right]\right)$

49. Solution

```
DSolve[{y'[x] == y[x] * Log[y[x]] + y[x] * E^x, y[0] == 1}, y[x], x]
```

$\{\{y[x] \to e^{e^x x}\}\}$

50. Solution

```
Mod[Fibonacci[2006], 10]
```

3

51. Solution

```
Integrate[(E - 1) * Sqrt[Log[1 + E * x - x]] + E^(x^2), {x, 0, 1}]
```

e

52. Solution

```
FullSimplify[2 * Cos[Pi / 7]^3 - Cos[Pi / 7]^2 - Cos[Pi / 7]]
```

$-\frac{1}{4}$

53. Solution

```
Sqrt[7 + Sqrt[40]] // FullSimplify
```

$\sqrt{2} + \sqrt{5}$

54. Solution

```
Module[{TriangleQ},
 TriangleQ[{s1_, s2_, s3_}] :=
  If[s1 + s2 > s3 && s2 + s3 > s1 && s3 + s1 > s2, True, False];
 Length[Select[IntegerPartitions[1994, {3}], TriangleQ]]]
```

82 834

55. Solution

```
Select[{Mean[Table[i^2, {i, 1, #}]], #} & /@ Range[2, 500] ,
   IntegerQ[First[#]^(1 / 2)] &][[1]][[2]]
```

337

56. Solution

```
Module[{special},
 special[n_] := Module[{qr},
   qr = QuotientRemainder[n, 1000];
   If[qr[[1]] + 1 == qr[[2]] && IntegerQ[n^(1 / 2)], True, False]
  ];
 Select[ Range[100 000, 999 999] , special[#] &]
]
```

{183 184, 328 329, 528 529, 715 716}

57. Solution

```
Length[Select[
  {1, x, x^2, x^3, x^4, x^5}.# & /@ Permutations[{1, 2, 3, 4, 5, 6, 7, 8}, {6}],
  PolynomialRemainder[#, x^2 - x + 1, x] == 0 &]]
```

```
300
```

58. Solution

```
512^3 + 675^3 + 720^3
FactorInteger[512^3 + 675^3 + 720^3]
```

```
815 012 603
```

```
{{229, 1}, {467, 1}, {7621, 1}}
```

59. Solution

```
Select[Range[1000, 70 000], Length[Divisors[#]] ≥ 101 &]
```

```
{50 400, 55 440, 60 480, 65 520, 69 300}
```

60. Solution

```
Select[Range[100, 999],
 If[Mod[#, 11] == 0 && Total[#^2 & /@ IntegerDigits[#]] == Quotient[#, 11],
   True, False] &]
```

```
{550, 803}
```

61. Solution

```
Numerator[Total[(-1)^(# + 1) * (1 / #) & /@ Range[1, 1319]]]
Mod[Numerator[Total[(-1)^(# + 1) * (1 / #) & /@ Range[1, 1319]]] , 1979]
```

```
9 712 435 370 176 457 211 789 032 112 275 705 035 436 058 023 056 668 172 162 691 122 769 643 ⋱
  536 134 963 388 887 418 503 518 212 227 707 181 887 399 284 421 324 209 991 580 261 372 180 ⋱
  881 066 042 267 340 013 822 543 021 630 993 790 301 871 917 306 672 479 003 742 347 731 906 ⋱
  894 502 206 611 684 973 377 349 822 140 727 866 748 793 472 314 555 146 081 954 133 359 219 ⋱
  693 224 842 345 716 201 562 838 113 110 488 664 229 990 443 258 508 595 332 395 129 617 286 ⋱
  143 580 898 566 908 128 232 436 187 466 371 871 641 095 608 257 859 832 669 859 587 159 077 ⋱
  972 843 255 741 143 946 005 144 546 720 084 104 126 426 401 254 209 483 858 435 525 146 637 ⋱
  078 869 452 056 788 708 944 608 960 016 712 180 409 515 668 301 909 686 056 363 654 039 823 ⋱
  351 264 466 923 197 720 153 813
```

```
0
```

62. Solution

The expression below is equal to $\frac{2n!}{n!\,n!}$which is an integer.

```
Product[(4 - 2/i), {i, 1, n}]
```

$$\frac{4^n \text{Gamma}\left[\frac{1}{2} + n\right]}{\sqrt{\pi}\ \text{Gamma}[1 + n]}$$

63. Solution

```
Select[Range[100, 1997], Mod[2^# + 2, #] == 0 &]
```

```
{946}
```

64. Solution

```
Total[Divisors[104 060 401]]
```

```
105 101 005
```

65. Solution

```
FactorInteger[1 280 000 401]
{{421, 1}, {3 040 381, 1}}
```

66. Solution

```
Select[Range[1000, 5000], Mod[2^33 - 2^19 - 2^17 - 1, #] == 0 &]
{1983}
```

67. Solution

```
FactorInteger[989 * 1001 * 1007 + 320]
{{991, 1}, {997, 1}, {1009, 1}}
```

68. Solution

```
Solve[n^5 == 133^5 + 110^5 + 84^5 + 27^5, n, Integers]
{{n → 144}}
```

69. Solution

```
Solve[n^13 == 21 982 145 917 308 330 487 013 369, n, Integers]
{{n → 89}}
```

70. Solution

```
Reduce[(x + y)^2 - 2 (x * y)^2 == 1 && x > 0 && y > 0, {x, y}, Integers]
(x == 1 && y == 2) || (x == 2 && y == 1)
```

71. Solution

```
Reduce[5 m^2 - 6 m * n + 7 n^2 == 1985, {m, n}, Integers]
False
```

72. Solution

```
Reduce[x^3 - 3 x * y^2 - y^3 == 1, {x, y}, Integers]
(x == -3 && y == 2) || (x == -1 && y == 1) || (x == 0 && y == -1) ||
  (x == 1 && y == -3) || (x == 1 && y == 0) || (x == 2 && y == 1)
```

73. Solution

```
Factor[x^8 + 98 x^4 + 1]
```

$$\left(1+4x+8x^2-4x^3+x^4\right)\left(1-4x+8x^2+4x^3+x^4\right)$$

74. Solution

```
Max[Table[If[(n^2 - m * n - m^2)^2 == 1, m^2 + n^2, 0], {m, 1, 1981}, {n, 1, 1981}]]
3 524 578
```

75. Solution

```
Length[Select[Range[1, 332], IntegerDigits[2^#][[1]] == 4 &]]
33
```

76. Solution

```
Mod[(RSolve[{u[n + 1] == 1996 u[n] + 1997 * u[n - 1], u[1] == 1, u[2] == 1}, u[n], n] /.
      n → 1997)[[1]][[1]][[2]], 3]
```

2

77. Solution

```
Sum[k * 2^(k - 1), {k, 1, n}]
```

$1 - 2^n + 2^n n$

78. Solution

```
RSolve[{u[n + 1] == u[n] / (1 + n * u[n]), u[0] == 1}, u[n], n]
```

$\left\{\left\{u[n] \to \frac{2}{2 - n + n^2}\right\}\right\}$

79. Solution

```
Sum[1 / Round[Sqrt[i]], {i, 1, 1980}]
```

88

80. Solution

```
Total[IntegerDigits[Sum[10^i - 1, {i, 1, 100}]]]
```

99

81. Solution

```
Sum[1 / ((2 i - 1) * (2 i) * (2 i + 1)), {i, 1, Infinity}]
```

$\frac{1}{2} (-1 + \text{Log}[4])$

82. Solution

```
FullSimplify[Limit[(1 / n^2) Product[(n^2 + i^2)^(1 / n), {i, 1, n}], n → Infinity]]
```

83. Solution

```
Sum[Fibonacci[k] / 3^k, {k, 1, Infinity}]
```

$\frac{3}{5}$

84. Solution

```
RSolve[{u[n + 1] - u[n] - u[n - 1] + u[n - 2] - 1 == 0,
    u[0] == 0, u[1] == 1, u[2] == 2}, u[n], n] /. n → 2004
```

{{u[2004] → 1 005 006}}

85. Solution

```
(-1)^(Sum[Floor[n / k], {k, 1, n}] /. n → 2004)
```

1

86. Solution

```
Integrate[Log[Sin[x]], {x, 0, Pi}]
```

$-\pi \text{Log}[2]$

87. Solution

```
FixedPoint[Total[IntegerDigits[#]] &, 2^2006]
```

4

88. Solution

```
Sum[(i^2 + 1) * Factorial[i], {i, 1, n}]
```

n (1 + n) !

89. Solution

```
Nest[Total[#^2 & /@ IntegerDigits[#]] & , 2006, 2007]
```

42

90. Solution

```
IntegerQ[Sum[n^2009 / 2^n, {n, 1, Infinity}]]
```

True

91. Solution

```
Sum[Sum[1 / k, {k, 1, n + 1}] / (n * (n + 1)), {n, 1, Infinity}]
```

2

92. Solution

```
Nest[(# - 1) / (# + 1) & , 2015, 2015]
```

$-\frac{1008}{1007}$

93. Solution

```
Integrate[Log[x] / (x^2 + 9), {x, 0, Infinity}]
```

$\frac{1}{6} \pi \text{Log}[3]$

94. Solution

```
Sum[1 / (Sqrt[k] + Sqrt[k + 1]), {k, 1, n}] /. n → 9999
```

99

95. Solution

```
Flatten[ConstantArray[#, #] & /@ Range[1, 2013]][[2013]]
```

63

96. Solution

```
FullSimplify[Sum[Binomial[k, n] * 2^(2 n - k), {k, n, 2 n}]]
```

4^n

97. Solution

```
Last[IntegerDigits[Floor[10^20 000 / (10^100 + 3)]]]
```

3

98. Solution

```
Hold[Sum[k / Factorial[k + 1], {k, 1, n}] // TraditionalForm]
```

$$\text{Hold}\left[\sum_{k=1}^{n} \frac{k}{(k+1)!}\right]$$

99. Solution

```
Sum[Binomial[n + k, k] 2^-k, {k, 0, n}]
```

2^n

100. Solution

```
Length[Select[Range[1, 2001], FixedPoint[Total[IntegerDigits[#]] &, #] == 9 &] ]
```

222

101. Solution

```
Solve[Sum[k * x^k, {k, 1, Infinity}] == 20, x, Reals]
```

$\left\{\left\{x \to \frac{4}{5}\right\}, \left\{x \to \frac{5}{4}\right\}\right\}$

102. Solution

```
Total[Select[Range[10, 99],
  Take[IntegerDigits[#^2], -2] == Take[IntegerDigits[#^3], -2] &] ]
```

772

103. Solution

```
Integrate[(1 / 3) * Floor[2 x^3 - 2 Floor[x^3]], {x, -1.5, 1.5}]
```

0.5

104. Solution

```
Module[{M},
 M = Partition[Range[1, 100], 10];
 Length[Select[Flatten[M + Transpose[M]], # == 101 &]]]
```

10

105. Solution

```
Floor[10^2017 / Sum[(10^i - 1) / 9, {i, 1, n}] /. n → 2014]
```

8100

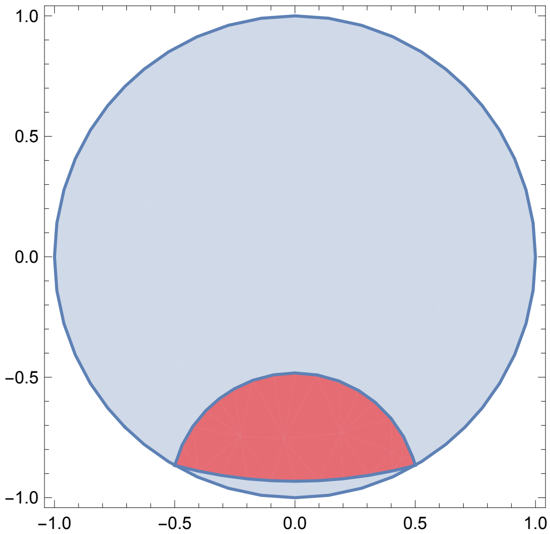
106. Solution

```
Mod[Nest[#^2 - 2 &, 18, 24], 89]
```

47

107. Solution

```
Module[{a, b, c, ix, iy, cur, R, C, RC},
 a = EuclideanDistance[{x1, y1}, {1 / 2, -√3 / 2}];
 b = 1;
 c = EuclideanDistance[{-1 / 2, -√3 / 2}, {x1, y1}];
 ix = (a * (-1 / 2) + b * x1 + c * (1 / 2)) / (a + b + c);
 iy = (a * (-√3 / 2) + b * y1 + c * (-√3 / 2)) / (a + b + c);
 cur = Eliminate[x == ix && y == iy && x1^2 + y1^2 == 1, {x1, y1}] /.
   (lhs_) == (rhs_) → lhs - rhs;
 R = ImplicitRegion[Evaluate[cur] > 0 , {{x, -1, 1}, {y, -1, 1}}];
 C = ImplicitRegion[x^2 + y^2 ≤ 1 , {{x, -1, 1}, {y, -1, 1}}];
 RC = RegionIntersection[R, C];
 Print[Show[RegionPlot[C],
   RegionPlot[RC, PlotStyle → Directive[Opacity[0.5], Red]]]];
 FullSimplify[RegionMeasure[RC]]]
```



$-1 + \pi - \frac{\pi}{\sqrt{3}}$

108. Solution

```
Round[Log[Times @@ (#[[2]] + 1 & /@ FactorInteger[Factorial[2014]])]]
```

439

109. Solution

```
Sum[Log[2, (1 - (1 / n)) / (1 - (1 / (n + 1)))], {n, 2, Infinity}]
```

-1

110. Solution

```
Simplify[2 Cos[10 Degree] + Sin[100 Degree] + Sin[1000 Degree] + Sin[10 000 Degree]]
```

Cos[10 °]

111. Solution

```
Fold[Dot, {{1, 0}, {0, 1}}, Table[{{1, i}, {0, 1}}, {i, 1, 99, 2}]] // MatrixForm
```

$\begin{pmatrix} 1 & 2500 \\ 0 & 1 \end{pmatrix}$

112. Solution

```
Solve[a^2 - b^2 - c^2 - d^2 == c - b - 2 && 2 * a * b == a - d - 32 &&
  2 * a * c == 28 - a - d && 2 * a * d == b + c + 31, {a, b, c, d}, Integers]
```

```
{{a → 5, b → -3, c → 2, d → 3}}
```

113. Solution

```
Solve[1 / 2015 == a / 5 + b / 13 + c / 31 && 0 ≤ a && a < 5 && 0 ≤ b && b < 13,
 {a, b, c}, Integers]
```

```
{{a → 2, b → 12, c → -41}}
```

114. Solution

```
Length[Select[Range[1, 1 000 000],
  IntegerQ[#^(1 / 2)] == True && IntegerQ[#^(1 / 3)] == False &]]
```

```
990
```

115. Solution

```
Module[{test},
 test[l_List] := Length[l] == 2 && l[[1]][[2]] == 1 && l[[2]][[2]] == 1 ;
 Length[Select[Range[1, 1 000 000], test[FactorInteger[#]] &]]]
```

```
209 867
```

116. Solution

```
Limit[((x - 1) (x - 2) (x - 3) (x - 4)) /
  ((x - 1) (x - (1 / 2)) (x - (1 / 3)) (x - (1 / 4))), x → 1]
```

```
-24
```

117. Solution

```
Sum[Cos[i * Pi / 18], {i, 2, 34, 2}]
```

```
-1
```

118. Solution

```
Module[{n = 1, i = 1},
 While[n < 2008,
  If[ MemberQ[IntegerDigits[i], 2], i++, i++; n++]];
 i]
```

```
3781
```

119. Solution

```
Take[IntegerDigits[17^17], -2]
```

```
{7, 7}
```

120. Solution

```
Coefficient[(x + 1)^6 * Sum[x^i, {i, 0, 6}], x, 6]
```

```
64
```

121. Solution

```
Total[Solve[Abs[x + 2] ≤ 10, x, Integers] /. {x → (a_)} → a]
```

```
-42
```

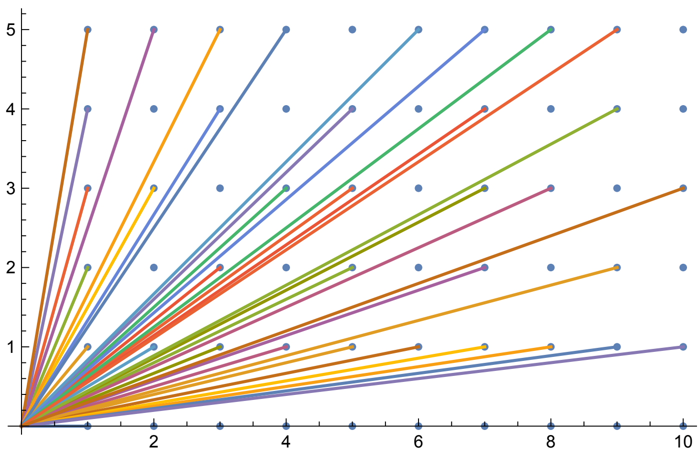
122. Solution

```
With[{n = 24 * 35 * 46 * 57},
 Max @@ Select[Range[1, Round[Sqrt[n]]], Mod[n, #^2] == 0 &]]
```

```
12
```

123. Solution

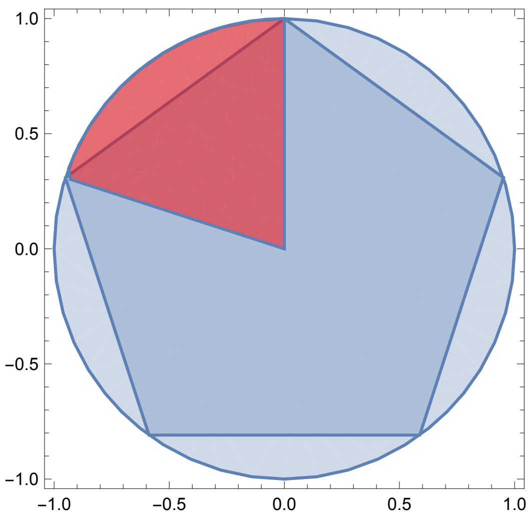
```
Module[{latpts, xl = 10, yl = 5},
 latpts[{x1_, y1_}, {x2_, y2_}] := Which[x1 == x2, Solve[
     (y2 - y1) * (x - x1)  == 0 && Min[y1, y2] < y <  Max[y1, y2], {x, y}, Integers],
    y1 == y2, Solve[ (x2 - x1) * (y - y1)  == 0 && Min[x1, x2] < x <  Max[x1, x2],
     {x, y}, Integers],
    x1 ≠ x2 && y1 ≠ y2,
    Solve[(x2 - x1) * (y - y1)  -  (y2 - y1) * (x - x1)  == 0 &&
      Min[x1, x2] < x  <  Max[x1, x2] && Min[y1, y2] < y <  Max[y1, y2],
     {x, y}, Integers]]  /. {x → a_, y → b_} → {a, b} ;
 Print[Show[ListPlot[Flatten[Table[{i, j}, {i, 1, xl}, {j, 0, yl}], 1]],
   ListLinePlot[{{0, 0}, #} & /@
     Map[#[[1]] & , Select[Flatten[Table[{{i, j}, latpts[{0, 0}, {i, j}]},
         {i, 1, xl}, {j, 0, yl}], 1], Length[#[[2]]] == 0 &]]]]];
 Length[Select[Flatten[Table[{{i, j}, latpts[{0, 0}, {i, j}]},
     {i, 1, xl}, {j, 0, yl}], 1], Length[#[[2]]] == 0 &]]
]
```




```
36
```

124. Solution

```
Module[{CP, C, P, R, RI},
 CP = CirclePoints[5];
 P  = Polygon[CP];
 C  = ImplicitRegion[x^2 + y^2 ≤ 1, {x, y}];
 R = ImplicitRegion[{EuclideanDistance[{x, y}, CP[[1]]] ≥
      Max @@ Table[EuclideanDistance[{x, y} , CP[[i]]], {i, 2, 5}]},
    {{x, -1, 1}, {y, -1, 1}}];
 RI  = RegionIntersection[R, C];
 Print[Show[RegionPlot[C], RegionPlot[P],
    RegionPlot[RI, PlotStyle → Directive[Opacity[0.5], Red]]]];
 FullSimplify[RegionMeasure[RI]]]
```



$\frac{\pi}{5}$

125. Solution

```
Module[{n = 1, i = 2},
 While[n < 21,
  If[ PrimeQ[(2^i) - 1], n++; i++, i++]];
 i - 1]
```

4423

126. Solution

```
Factor[x^4 + 2 x^3 + 2 x^2 + 2 x + 1]
Solve[x^4 + 2 x^3 + 2 x^2 + 2 x + 1 == 0, x]
```

$(1+x)^2\,(1+x^2)$

$\{\{x\to -1\},\{x\to -1\},\{x\to -i\},\{x\to i\}\}$

127. Solution

```
Factor[x^9 + (9 / 8) x^6 + (27 / 64) x^3 - x + 219 / 512]
FullSimplify[Solve[x^9 + (9 / 8) x^6 + (27 / 64) x^3 - x + 219 / 512 == 0, x, Reals]]
```

$\frac{1}{512}(-1+2x)\left(-3+2x+4x^2\right)\left(73+24x+64x^2+48x^3+64x^4+64x^6\right)$

$\left\{\left\{x\to\frac{1}{2}\right\},\left\{x\to\frac{1}{4}\left(-1-\sqrt{13}\right)\right\},\left\{x\to\frac{1}{4}\left(-1+\sqrt{13}\right)\right\}\right\}$

128. Solution

```
BitString[n_] := If[n == 1, {{0}, {1}},
   Join[Append[#, 0] & /@ BitString[n - 1], Append[#, 1] & /@ BitString[n - 1]]];
Length[Select[Dot[{1, 3, 9, 27, 81, 243, 729}, #] & /@ BitString[7], 1 ≤ # ≤ 1000 &]]
```

105

129. Solution

```
Total[Length[Divisors[#]] & /@ Range[1, 2012]]
```

15 612

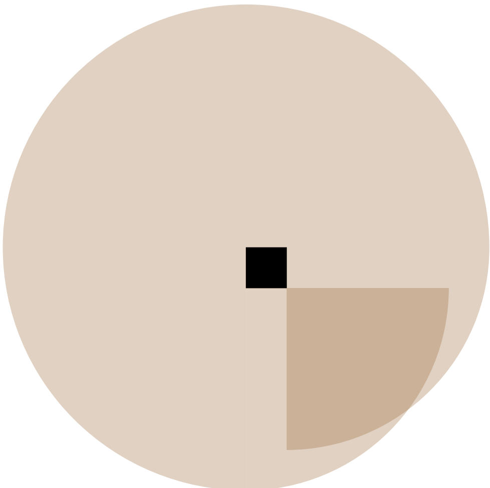
130. Solution

```
PolynomialMod[x + x^9 + x^25 + x^49 + x^81, x^3 - x]
```

5 x

131. Solution

```
Module[{S, Q3, R1, R2},
 S = Rectangle[{0, -1}];
 Q3 = Disk[{0, 0}, 6, {0, 3 * Pi / 2}];
 R1 = Disk[{0, -1}, 5, {3 Pi / 2, 2 Pi}];
 R2 = Disk[{1, 0}, 5, {3 Pi / 2, 2 Pi}];
 Print[Show[Graphics[{Black, S, Opacity[0.3, Brown],
     Q3, Opacity[0.3, Brown], R1, Opacity[0.3, Brown], R2}]]];
 Print[FullSimplify[RegionMeasure[Q3] + RegionMeasure[RegionUnion[R1, R2]]]];
]
```



$3 + \frac{133\pi}{4} + \frac{25}{2}\text{ArcSin}\left[\frac{7}{25}\right]$

132. Solution

```
Minimize[EuclideanDistance[{x, y}, {3, 0}]^2 +
  EuclideanDistance[{x, y}, {0, 4}]^2 + EuclideanDistance[{x, y}, {0, 0}]^2,
 {x, y} ∈ Triangle[{{0, 0}, {3, 0}, {0, 4}}]]
```

$\left\{\frac{50}{3}, \left\{x \to 1, y \to \frac{4}{3}\right\}\right\}$

133. Solution

```
Module[{AllPoss},
 AllPoss[l_List, n_] := If[n == 1, {#} & /@ l,
   Flatten[Function[{pos}, Append[pos, #] & /@ l ] /@ AllPoss[l, n - 1], 1]];
 Print[AllPoss[{-1, 0, 1}, 1]];
 Length[Select[Subsets[AllPoss[{-1, 0, 1}, 3], {3}],
   Norm[Cross[#[[1]] - #[[3]], #[[1]] - #[[2]]]] == 0 &]]]
```

```
{{-1}, {0}, {1}}
```

```
49
```

134. Solution

```
Length[Select[Range[2, 100], Module[{l},
    l = FactorInteger[#];
    Times @@ Map[#[[1]] &, l] < 10] &]]
```

```
23
```

135. Solution

```
Length[Select[Range[10, 99],
  IntegerQ[(# + FromDigits[Reverse[IntegerDigits[#]]])^(1 / 2)] &]]
```

```
8
```

136. Solution

```
Length[Select[Range[1000, 9999], Module[{l},
    l = IntegerDigits[#];
    l[[1]] + l[[3]] == 2 * l[[2]] && l[[2]] + l[[4]] == 2 * l[[3]]] &]]
```

```
30
```

137. Solution

```
Length[Select[Range[10 000, 99 999], Module[{l},
    l = IntegerDigits[#];
    l[[1]] == 3 && l[[1]] > l[[2]] > l[[3]] && l[[3]] < l[[4]] < l[[5]]] &]]
```

```
100
```

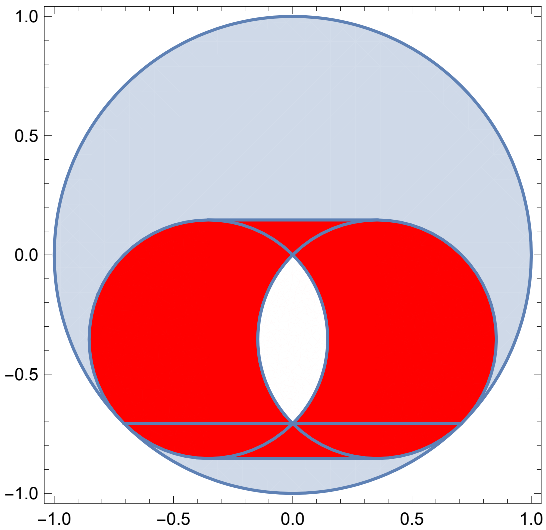
138. Solution

```
Select[Range[2, 100], Length[Divisors[#]] == 6 &][[1]]
```

```
12
```

139. Solution

```
Module[{A, B, C, M, N, RM, RN, RE, Q, R, P, S},
 A = {-1/√2, -1/√2};
 B = {1/√2, -1/√2};
 C = {x, y};
 M = (A + C) / 2;
 N = (B + C) / 2;
 P = (A + {0, 1}) / 2;
 Q = (B + {0, 1}) / 2;
 R = (A + {0, -1}) / 2;
 S = (B + {0, -1}) / 2;
 RE = Rectangle[R, Q];
 RM = Eliminate[x1 == M[[1]] && y1 == M[[2]] && x^2 + y^2 == 1, {x, y}] /.
   (lhs_) == (rhs_) → lhs - rhs;
 RN = Eliminate[x1 == N[[1]] && y1 == N[[2]] && x^2 + y^2 == 1, {x, y}] /.
   (lhs_) == (rhs_) → lhs - rhs;
 RM = ImplicitRegion[Evaluate[RM ≥ 0], {{x1, -1, 1}, {y1, -1, 1}}];
 RN = ImplicitRegion[Evaluate[RN ≥ 0], {{x1, -1, 1}, {y1, -1, 1}}];
 Print[Show[RegionPlot[x^2 + y^2 <= 1, {x, -1, 1}, {y, -1, 1}],
   RegionPlot[RE, PlotStyle → Directive[Red]], RegionPlot[RM,
    PlotStyle → Directive[Red]], RegionPlot[RN, PlotStyle → Directive[Red]],
   RegionPlot[RegionIntersection[RM, RN], PlotStyle → White],
   RegionPlot[Line[{A, B}]]]];
 Simplify[RegionMeasure[BooleanRegion[Or, {RM, RN, RE}]] -
   RegionMeasure[RegionIntersection[RM, RN]]]
]
```



$$\frac{1}{8}\left(2+4\sqrt{2}+\pi\right)$$

140. Solution

```
Module[{A},
 A = {0, 1};
 FullSimplify[Area[Triangle[{A, {x1, y1}, {-x1, y1}}]]
   /. Solve[{EuclideanDistance[{x1, y1}, A] == 2 x1, x1^2/4 + y1^2 == 1}, {x1, y1}]]
]
```

$$\left\{\frac{192\sqrt{3}}{169}, 0\right\}$$

141. Solution

```
Integrate[Sqrt[(4 - x) / x] - Sqrt[x / (4 - x)], {x, 0, 2}]
```

4

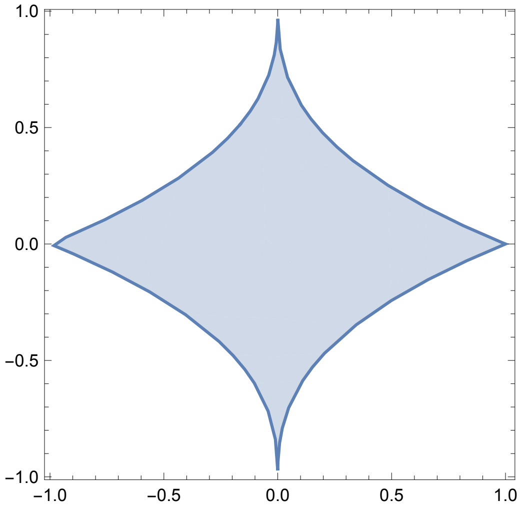
142. Solution

```
Simplify[Sum[1 / (2^i * i^2), {i, 1, Infinity}]]
```

$\frac{1}{12}\left(\pi^2 - 6\,\text{Log}[2]^2\right)$

143. Solution

```
RegionPlot[ImplicitRegion[x^(2 / 5) + Abs[y] ≤ 1, {{x, -2, 1}, {y, -1, 1}}]]
Integrate[1 - x^(2 / 5), {x, 0, 1}] * 4
```



$\frac{8}{7}$

144. Solution

```
Integrate[(1 + x^2) / (1 + 2^x), {x, -2, 2}]
```

$\frac{14}{3}$

145. Solution

```
Limit[x - x^2 Log[(1 + x) / x], x → Infinity]
```

$\frac{1}{2}$

146. Solution

```
Sort[FromDigits[#] & /@ Permutations[Range[1, 7]]][[2013]]
```

3 657 214

147. Solution

```
FullSimplify[#] & /@
 DeleteDuplicates[z + z^3 + z^4 + z^9 + z^10 + z^12 /. Solve[z^13 == 1, {z}]]
```

$\left\{6, \frac{1}{2}\left(-1 - \sqrt{13}\right), \frac{1}{2}\left(-1 + \sqrt{13}\right)\right\}$

148. Solution

```
Simplify[Total[
  Solve[(4 x^2 + 15 x + 17) (2 x^2 + 5 x + 13) - (x^2 + 4 x + 12) (5 x^2 + 16 x + 18) == 0,
    x, Reals] /. {x_ → a_} → a ]]
```

$-\frac{11}{3}$

149. Solution

```
Max @@ (Solve[x^4 - x^3 - 5 x^2 + 2 x + 6 == 0, x] /. {x_ -> a_} → a)
```

$\frac{1}{2}\left(1+\sqrt{13}\right)$

150. Solution

```
Solve[(20 x + (20 x + 13) ^ (1 / 3)) ^ (1 / 3) == 13, x, Reals]
```

$\left\{\left\{x \to \frac{546}{5}\right\}\right\}$

151. Solution

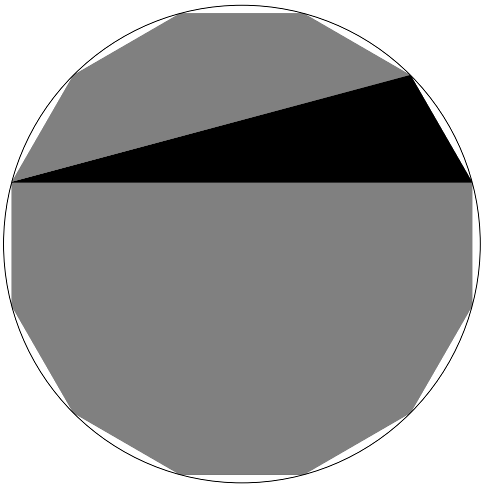
```
Module[{a = -√3 + √5 + √7, b = √3 - √5 + √7, c = √3 + √5 - √7},
 Simplify[a^4 / ((a - b) (a - c)) + b^4 / ((b - c) (b - a)) + c^4 / ((c - a) (c - b))]]
```

30

152. Solution

```
Module[{T, C, r},
 T = SSSTriangle[√2, 3 + √3, 2 √2 + √6];
 C = Circumsphere @@ T;
 r = C[[2]];
 Print[FullSimplify /@ {ArcSin[√2 / (2 r)] / Pi ,
    ArcSin[(3 + √3) / (2 r)] / Pi,  ArcSin[(2 √2 + √6) / (2 r)] / Pi}];
 Print[Show[Graphics[{Gray, Polygon[CirclePoints[C[[1]], r, 12]]}],
   Graphics[{T, C}]]];
 FullSimplify[Area[Polygon[CirclePoints[C[[1]], r, 12]]]]]
```

$\left\{\frac{1}{12}, \frac{1}{3}, \frac{5}{12}\right\}$



$6\left(2+\sqrt{3}\right)$

153. Solution

```
Sum[1 / (m * n * (m + n + 1)), {m, 1, Infinity}, {n, 1, Infinity}]
```

2

154. Solution

```
Length[FrobeniusSolve[{15, 21, 35}, 525]]
```

21

155. Solution

```
Sum[1 / (n^3 * (n + 1)^3), {n, 1, Infinity}]
```

$10 - \pi^2$

156. Solution

```
Simplify[
 Length[
   Select[
    Flatten[Table[{i, j, k, l}, {i, 1, 8}, {j, 1, 8}, {k, 1, 8}, {l, 1, 8}], 3],
    Function[{p},
     Not[(p[[1]] == p[[3]] && p[[2]] == p[[4]])] &&
      (Abs[p[[3]] - p[[1]] ] == 0 || Abs[p[[2]] - p[[4]] ] == 0 ||
        (Abs[p[[3]] - p[[1]] ] == Abs[p[[2]] - p[[4]] ] ))]]] / (64 * 63)]
```

$\frac{13}{36}$

157. Solution

```
Sum[((7 n + 32) * 3^n) / (n * (n + 2) * 4^n), {n, 1, Infinity}]
```

$\frac{33}{2}$

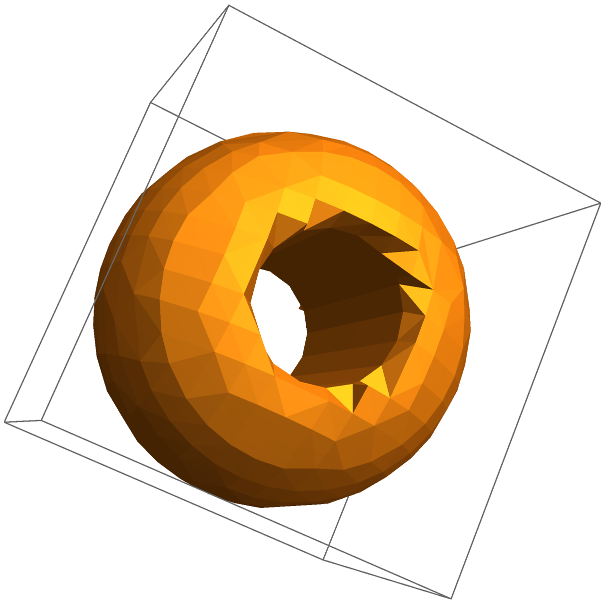
158. Solution

```
Minimize[
 √2 * EuclideanDistance[{x, y}, {1, 0}] + EuclideanDistance[{x, y}, {0, 0}] +
  EuclideanDistance[{x, y}, {0, 1}] , {x, y} ∈ Rectangle[{0, 0}]]
```

$\{\sqrt{5}, \{x \to \frac{2}{5}, y \to \frac{1}{5}\}\}$

159. Solution

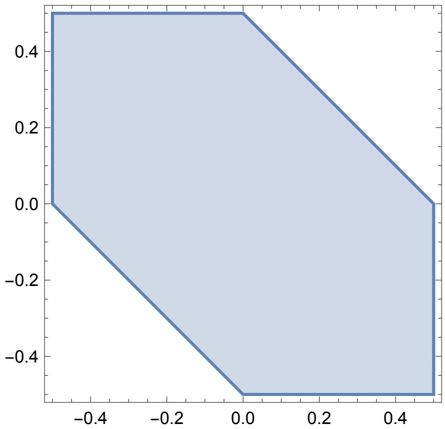
```
Module[{S, C},
 S = Ball[{0, 0, 0}, 13];
 C = Cylinder[{{0, 0, -14}, {0, 0, 14}}, 5];
 Print[RegionPlot3D[RegionDifference[S, C]]];
 ((RegionMeasure[S] - RegionMeasure[RegionIntersection[S, C]]) * 3 / (4 Pi)) ^
  (1 / 3)
]
```



12

160. Solution

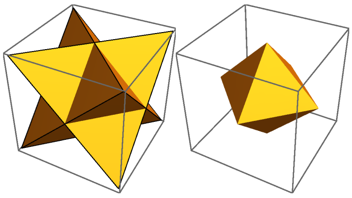
```
Module[{R},
 R = ImplicitRegion[Abs[x] + Abs[y] + Abs[x + y] ≤ 1, {x, y}];
 Print[RegionPlot[R]];
 Area[R]
]
```



$\frac{3}{4}$

161. Solution

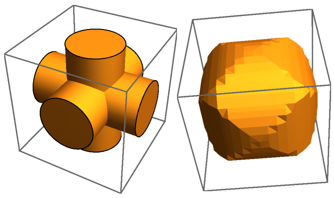
```
Module[{R1, R2, R},
 R1 = Tetrahedron[{{1, 1, 1}, {-1, -1, 1}, {-1, 1, -1}, {1, -1, -1}}];
 R2 = Tetrahedron[{{-1, -1, -1}, {1, 1, -1}, {1, -1, 1}, {-1, 1, 1}}];
 R = RegionIntersection[R1, R2];
 Print[Show[RegionPlot3D[R1], RegionPlot3D[R2]], Show[RegionPlot3D[R]]];
 RegionMeasure[R] / RegionMeasure[R1]
]
```



$\frac{1}{2}$

162. Solution

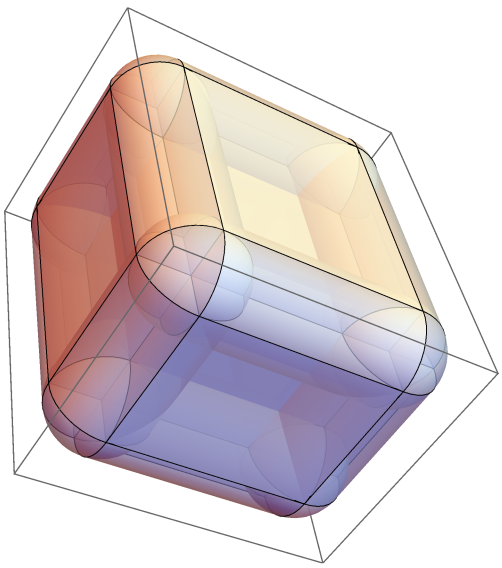
```
Module[{C1, C2, C3, R},
 C1 = Cylinder[{{0, 0, -2}, {0, 0, 2}}, 1];
 C2 = Cylinder[{{0, -2, 0}, {0, 2, 0}}, 1];
 C3 = Cylinder[{{-2, 0, 0}, {2, 0, 0}}, 1];
 R = RegionIntersection[C1, C2, C3];
 Print[Show[RegionPlot3D[C1], RegionPlot3D[C2], RegionPlot3D[C3]],
  Show[RegionPlot3D[R]]];
 RegionMeasure[R]
]
```



$-8\left(-2+\sqrt{2}\right)$

163. Solution

```
Module[{CS, SS, CYS},
 CS = Cuboid @@@ {{{0, 0, 0}, {5, 4, 3}},
    {{-1, 0, 0}, {0, 4, 3}},
    {{0, -1, 0}, {5, 0, 3}},
    {{5, 0, 0}, {6, 4, 3}},
    {{0, 4, 0}, {5, 5, 3}},
    {{0, 0, 3}, {5, 4, 4}},
    {{0, 0, -1}, {5, 4, 0}}
   };
 SS = Ball /@ {{0, 0, 0}, {5, 0, 0}, {5, 4, 0}, {0, 4, 0},
    {0, 0, 3}, {5, 0, 3}, {5, 4, 3}, {0, 4, 3}};
 CYS = Cylinder @@@ {
    {{{0, 0, 0}, {5, 0, 0}}, 1},
    {{{5, 0, 0}, {5, 4, 0}}, 1},
    {{{5, 4, 0}, {0, 4, 0}}, 1},
    {{{0, 4, 0}, {0, 0, 0}}, 1},
    {{{0, 0, 3}, {5, 0, 3}}, 1},
    {{{5, 0, 3}, {5, 4, 3}}, 1},
    {{{5, 4, 3}, {0, 4, 3}}, 1},
    {{{0, 4, 3}, {0, 0, 3}}, 1},
    {{{0, 0, 0}, {0, 0, 3}}, 1},
    {{{5, 0, 0}, {5, 0, 3}}, 1},
    {{{5, 4, 0}, {5, 4, 3}}, 1},
    {{{0, 4, 0}, {0, 4, 3}}, 1}
   };
 Print[Graphics3D[{Opacity[0.7], CS, CYS, SS}]];
 Simplify[RegionMeasure[RegionUnion @@ Join[SS, CYS, CS]] - 6]
]
```



$148 + \frac{40\pi}{3}$

164. Solution

```
FullSimplify[Integrate[x^3 / (E^x - 1), {x, 0, Infinity}]]
```

$\frac{\pi^4}{15}$

165. Solution

```
FullSimplify[Integrate[Sqrt[(1 + x) / (1 - x)], {x, -1, 1}]]
```

π

166. Solution

```
FullSimplify[Integrate[Log[x] / (1 + x^2), {x, 0, Infinity}]]
```

0

167. Solution

```
FullSimplify[Integrate[1 / (2 x + 3 (1 - x))^2, {x, 0, 1}]]
```

$\frac{1}{6}$

168. Solution

```
FullSimplify[Integrate[(x^4 * (1 - x)^4) / (1 + x^2), {x, 0, 1}]]
```

$\frac{22}{7} - \pi$

169. Solution

```
Solve[2 x^2 - 4 x y + 3 y^2 == 36 && 3 x^2 - 4 x y + 2 y^2 == 36, {x, y}]
```

{{x → -6, y → -6}, {x → -2, y → 2}, {x → 2, y → -2}, {x → 6, y → 6}}

170. Solution

```
Solve[5732 x + 2134 y + 2134 z == 7866 && 2134 x + 5732 y + 2134 z == 670 &&
  2134 x + 2134 y + 5732 z == 11 464 , {x, y, z}]
```

{{x → 1, y → -1, z → 2}}

171. Solution

```
Sum[(-1)^k (k / (4 k^2 - 1)), {k, 1, n}]
```

$\frac{-1 + (-1)^n - 2 n}{4 (1 + 2 n)}$

172. Solution

```
FullSimplify[Sum[Binomial[2 k, k] * 4^(-k), {k, 1, n}]]
```

$-1 + \frac{2 \text{Gamma}\left[\frac{3}{2} + n\right]}{\sqrt{\pi} \text{ Gamma}[1 + n]}$

173. Solution

```
Select[Range[100 000, 999 999],
 Sort[IntegerDigits[# * 2]] == Sort[IntegerDigits[#]] &&
   Sort[IntegerDigits[# * 3]] == Sort[IntegerDigits[#]] &&
   Sort[IntegerDigits[# * 4]] == Sort[IntegerDigits[#]] &&
   Sort[IntegerDigits[# * 5]] == Sort[IntegerDigits[#]] &&
   Sort[IntegerDigits[# * 6]] == Sort[IntegerDigits[#]] &]
```

{142 857}

174. Solution

```
DeleteDuplicates[IntegerDigits[
  FromDigits[ConstantArray[3, 666]] * FromDigits[ConstantArray[6, 666]]]]
```

{2, 1, 7, 8}

175. Solution

```
Select[Range[523001, 523999], Mod[#, 7] == 0 && Mod[#, 8] == 0 && Mod[#, 9] == 0 &]
```

{523152, 523656}

176. Solution

```
Map[FromDigits[#] &, #] & /@
 Select[ Partition[#, 3] & /@ Permutations[Range[1, 9]],
  FromDigits[#[[1]]] / FromDigits[#[[2]]] == 1 / 2 &&
    FromDigits[#[[2]]] / FromDigits[#[[3]]] == 2 / 3 &]
```

{{192, 384, 576}, {219, 438, 657}, {273, 546, 819}, {327, 654, 981}}

177. Solution

```
Total[Select[Range[1000, 9999], Module[{il},
    il = Sort[DeleteDuplicates[IntegerDigits[#]]];
    EvenQ[#] && Intersection[{0, 1, 2, 3, 4, 5}, il] == il ] &] ]
```

1769580

178. Solution

```
Apply[Join, IntegerDigits[#] & /@ Range[1, 90000]][[206788]]
```

7

179. Solution

```
FullSimplify[Sum[Cos[2 * i * Pi / (2 n + 1)], {i, 1, n}]]
```

$-\frac{1}{2}$

180. Solution

```
Sum[Binomial[n, k]^2, {k, 0, n}]
```

Binomial[2 n, n]

181.

```
Coefficient[(1 + x^2 - x^3)^1000, x^20] > Coefficient[(1 - x^2 + x^3)^1000, x^20]
```

True

182. Solution

```
Collect [FullSimplify[Sum[x^i, {i, 0, 100}] * Sum[x^i * (-1)^i, {i, 0, 100}]], x]
```

$1 + x^2 + x^4 + x^6 + x^8 + x^{10} + x^{12} + x^{14} + x^{16} + x^{18} + x^{20} + x^{22} + x^{24} + x^{26} + x^{28} + x^{30} + x^{32} + x^{34} + x^{36} + x^{38} + x^{40} + x^{42} + x^{44} + x^{46} + x^{48} + x^{50} + x^{52} + x^{54} + x^{56} + x^{58} + x^{60} + x^{62} + x^{64} + x^{66} + x^{68} + x^{70} + x^{72} + x^{74} + x^{76} + x^{78} + x^{80} + x^{82} + x^{84} + x^{86} + x^{88} + x^{90} + x^{92} + x^{94} + x^{96} + x^{98} + x^{100} + x^{102} + x^{104} + x^{106} + x^{108} + x^{110} + x^{112} + x^{114} + x^{116} + x^{118} + x^{120} + x^{122} + x^{124} + x^{126} + x^{128} + x^{130} + x^{132} + x^{134} + x^{136} + x^{138} + x^{140} + x^{142} + x^{144} + x^{146} + x^{148} + x^{150} + x^{152} + x^{154} + x^{156} + x^{158} + x^{160} + x^{162} + x^{164} + x^{166} + x^{168} + x^{170} + x^{172} + x^{174} + x^{176} + x^{178} + x^{180} + x^{182} + x^{184} + x^{186} + x^{188} + x^{190} + x^{192} + x^{194} + x^{196} + x^{198} + x^{200}$

183. Solution

```
Reduce[Abs[x + 1] - Abs[x] + 3 Abs[x - 1] - 2 Abs[x - 2] == x + 2, x, Reals]
```

$x == -2 \,||\, x \geq 2$

184. Solution

```
Reduce[Sqrt[x + 3 - 4 Sqrt[x - 1]] + Sqrt[x + 8 - 6 Sqrt[x - 1]] == 1, x, Reals]
```

$5 \leq x \leq 10$

185. Solution

```
Factor[a^10 + a^5 + 1]
```

$\left(1 + a + a^2\right)\left(1 - a + a^3 - a^4 + a^5 - a^7 + a^8\right)$

186. Solution

```
Reduce[Sqrt[a - Sqrt[a + x]] == x, x]
```

$$
\begin{aligned}
&\Bigg(0 == \frac{1}{2}\left(1 - \sqrt{-3+4a} - \sqrt{2}\sqrt{-1+2a-\sqrt{-3+4a}}\right) \,\&\& \\
&\quad 0 == \frac{1}{2}\left(-1 - \sqrt{-3+4a} - \sqrt{2}\sqrt{2a - \sqrt{2}\sqrt{-1+2a-\sqrt{-3+4a}}}\right) \,\&\& \\
&\quad x == \frac{1}{2}\left(-1 - \sqrt{-3+4a}\right)\Bigg) \,||\, \Bigg(0 == \frac{1}{2}\left(1 + \sqrt{-3+4a} - \sqrt{2}\sqrt{-1+2a+\sqrt{-3+4a}}\right) \,\&\& \\
&\quad 0 == \frac{1}{2}\left(-1 + \sqrt{-3+4a} - \sqrt{2}\sqrt{2a - \sqrt{2}\sqrt{-1+2a+\sqrt{-3+4a}}}\right) \,\&\& \\
&\quad x == \frac{1}{2}\left(-1 + \sqrt{-3+4a}\right)\Bigg) \,||\, \Bigg(0 == \frac{1}{2}\left(-1 + \sqrt{1+4a} - \sqrt{2}\sqrt{1+2a-\sqrt{1+4a}}\right) \,\&\& \\
&\quad 0 == \frac{1}{2}\left(1 - \sqrt{1+4a} - \sqrt{2}\sqrt{2a - \sqrt{2}\sqrt{1+2a-\sqrt{1+4a}}}\right) \,\&\&\, x == \frac{1}{2}\left(1 - \sqrt{1+4a}\right)\Bigg) \,|| \\
&\Bigg(0 == \frac{1}{2}\left(-1 - \sqrt{1+4a} - \sqrt{2}\sqrt{1+2a+\sqrt{1+4a}}\right) \,\&\& \\
&\quad 0 == \frac{1}{2}\left(1 + \sqrt{1+4a} - \sqrt{2}\sqrt{2a - \sqrt{2}\sqrt{1+2a+\sqrt{1+4a}}}\right) \,\&\&\, x == \frac{1}{2}\left(1 + \sqrt{1+4a}\right)\Bigg)
\end{aligned}
$$

187. Solution

```
FullSimplify[
 Solve[Area[SSSTriangle[x, x + d, x + 2 d]] == t && x > 0 && d > 0 && t > 0, x, Reals]]
```

$$\left\{\left\{x \to \text{ConditionalExpression}\left[-d + \sqrt{2 d^2 + \frac{2\sqrt{3 d^4 + 4 t^2}}{\sqrt{3}}},\right.\right.\right.$$
$$\left(0 < d < \frac{\sqrt{2}\sqrt{t}}{3^{1/4}} \mid\mid \frac{\sqrt{2}\sqrt{t}}{3^{1/4}} < d < \text{Root}\left[-16 t^2 + 9 \#1^4 \&, 2\right] \mid\mid\right.$$
$$\left.\left.\left. d > \text{Root}\left[-16 t^2 + 9 \#1^4 \&, 2\right]\right) \&\& t > 0\right]\right\},$$
$$\left\{x \to \text{ConditionalExpression}\left[-d + \sqrt{2 d^2 - \frac{2\sqrt{3 d^4 - 4 t^2}}{\sqrt{3}}},\right.\right.$$
$$\left.\left.\frac{\sqrt{2}\sqrt{t}}{3^{1/4}} < d < \text{Root}\left[-16 t^2 + 9 \#1^4 \&, 2\right] \&\& t > 0\right]\right\}, \{x \to$$
$$\text{ConditionalExpression}\left[\text{Root}\left[-9 d^4 + 16 t^2 - 12 d^3 \#1 + 6 d^2 \#1^2 + 12 d \#1^3 + 3 \#1^4 \&, 4\right],\right.$$
$$\left.\left.\left.\left(\frac{\sqrt{2}\sqrt{t}}{3^{1/4}} < d < \text{Root}\left[-16 t^2 + 9 \#1^4 \&, 2\right] \mid\mid d > \text{Root}\left[-16 t^2 + 9 \#1^4 \&, 2\right]\right) \&\& t > 0\right]\right\}\right\}$$

188. Solution

```
Select[PowersRepresentations[1986, 6, 2]]
```

{}

189. Solution

```
Solve[(x + y)^3 == z && (y + z)^3 == x && (z + x)^3 == y, {x, y, z}, Reals]
```

$$\left\{\{x \to 0, y \to 0, z \to 0\}, \left\{x \to -\frac{1}{2\sqrt{2}}, y \to -\frac{1}{2\sqrt{2}}, z \to -\frac{1}{2\sqrt{2}}\right\},\right.$$
$$\left.\left\{x \to \frac{1}{2\sqrt{2}}, y \to \frac{1}{2\sqrt{2}}, z \to \frac{1}{2\sqrt{2}}\right\}\right\}$$

190. Solution

```
Select[Range[0, 9],
 Mod[FromDigits[Join[ConstantArray[8, 50], {#}, ConstantArray[9, 50]]], 7] == 0 &]
```

{5}

191. Solution

```
FullSimplify[Fold[(#1) + (#2) + (#1) * (#2) &, Table[1/i, {i, 1, 10}]]]
```

10

192. Solution

```
Solve[n^2 + 9 n - 2 == (11 + n) k, n, Integers]
```

```
{{n → ConditionalExpression[-31, k == -34]},
 {n → ConditionalExpression[-21, k == -25]},
 {n → ConditionalExpression[-16, k == -22]},
 {n → ConditionalExpression[-15, k == -22]},
 {n → ConditionalExpression[-13, k == -25]},
 {n → ConditionalExpression[-12, k == -34]},
 {n → ConditionalExpression[-10, k == 8]}, {n → ConditionalExpression[-9, k == -1]},
 {n → ConditionalExpression[-7, k == -4]}, {n → ConditionalExpression[-6, k == -4]},
 {n → ConditionalExpression[-1, k == -1]}, {n → ConditionalExpression[9, k == 8]}}
```

```
(InterpolatingPolynomial[Table[{j, Fibonacci[j]}, {j, 992, 1982}], x] /.
   x → 1983) - Fibonacci[1983]
```

```
-1
```

193. Solution

```
Module[{P},
 P = PolynomialMod[a x^17 + b x^16 + 1, x^2 - x - 1];
 Solve[Coefficient[P, x] == 0 && (P /. x → 0) == 0, {a, b}, Integers]
]
```

```
{{a → 987, b → -1597}}
```

194. Solution

```
Select[Range[1, 90], PolynomialMod[x^13 + x + 90, x^2 - x + #] == 0 &]
```

```
{2}
```

195. Solution

```
Sum[((-1)^k / (1991 - k)) * Binomial[1991 - k, k], {k, 0, 995}]
```

$$\frac{1}{1991}$$

196. Solution

```
Minimize[Area[Triangle[{{0, -10}, {2, 0}, {x, x^2}}]], x]
```

$$\left\{\frac{15}{4}, \left\{x \to \frac{5}{2}\right\}\right\}$$

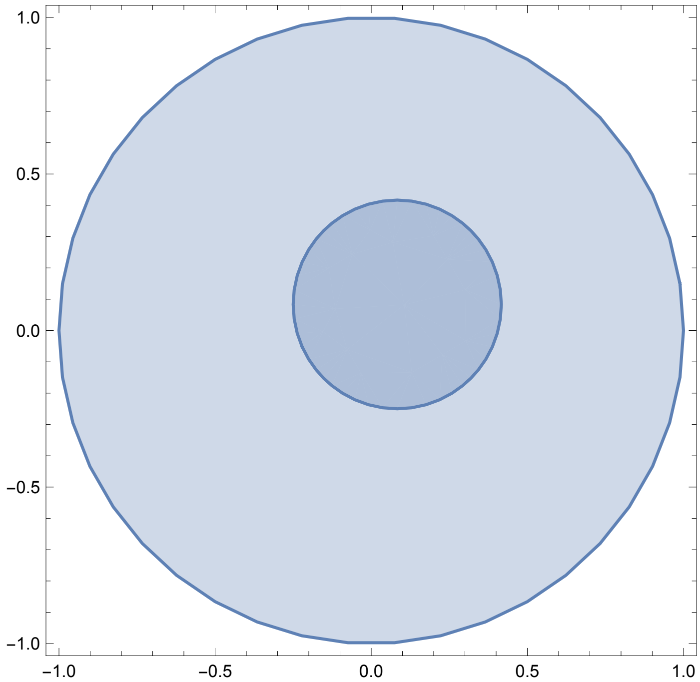
197. Solution

$$\text{Integrate}\left[\left(x^6 + x^3\right)\sqrt[3]{x^3 + 2},\ x\right]$$

$$\frac{1}{8} x^4 \left(2 + x^3\right)^{4/3}$$

198. Solution

```
Module[{R},
 R = ImplicitRegion[Evaluate[Eliminate[x1 == (a + x) / 3 && y1 == (b + y) / 3 &&
       x^2 + y^2 == 1, {x, y}] /. lhs_ == rhs_ → lhs - rhs ≥ 0], {x1, y1}];
 Print[Show[RegionPlot[Disk[{0, 0}, 1]], RegionPlot[R /. {a → 1 / 4, b → 1 / 4} ]]];
 RegionMeasure[R]
]
```



$$\frac{\pi}{9}$$

199. Solution

```
FullSimplify[Solve[x^4 - 6 x^2 y^2 + y^4 == 1 && 4 x^3 y - 4 x y^3 == 1, {x, y}, Reals]]
```

$$\left\{\left\{x \to -\frac{\left(1+3\sqrt{2}+2\sqrt{2\left(2+\sqrt{2}\right)}\right)^{1/4}}{2^{3/4}},\ y \to -\frac{\left(1+3\sqrt{2}-2\sqrt{2\left(2+\sqrt{2}\right)}\right)^{1/4}}{2^{3/4}}\right\},\right.$$

$$\left\{x \to \left(\frac{1}{8}+\frac{3}{4\sqrt{2}}+\frac{1}{2}\sqrt{1+\frac{1}{\sqrt{2}}}\right)^{1/4},\ y \to \frac{\left(1+3\sqrt{2}-2\sqrt{2\left(2+\sqrt{2}\right)}\right)^{1/4}}{2^{3/4}}\right\},$$

$$\left\{x \to -\frac{\left(1+3\sqrt{2}-2\sqrt{2\left(2+\sqrt{2}\right)}\right)^{1/4}}{2^{3/4}},\ y \to \left(\frac{1}{8}+\frac{3}{4\sqrt{2}}+\frac{1}{2}\sqrt{1+\frac{1}{\sqrt{2}}}\right)^{1/4}\right\},$$

$$\left.\left\{x \to \frac{\left(1+3\sqrt{2}-2\sqrt{2\left(2+\sqrt{2}\right)}\right)^{1/4}}{2^{3/4}},\ y \to -\frac{\left(1+3\sqrt{2}+2\sqrt{2\left(2+\sqrt{2}\right)}\right)^{1/4}}{2^{3/4}}\right\}\right\}$$

200. Solution

```
Sum[1/(2 i^2 - i), {i, 1, Infinity}]
```

Log[4]

201. Solution

```
Sum[((-1)^k/(2 n + 2 k + 1)) * Binomial[n, k], {k, 0, n}]
```

$$\frac{2^{-2 n} n \sqrt{\pi} (-1+2 n) !}{\left(\frac{1}{2} (1+4 n)\right) !}$$

202. Solution

```
Solve[((1 - √2 + √3)/(1 + √2 - √3)) == (√x + √y)/2, {x, y}, Integers]
```

{{x → 2, y → 6}, {x → 6, y → 2}}

203. Solution

```
Simplify[InterpolatingPolynomial[Table[1/k, {k, 1, 9}], x]] /. x → 10
```

$$\frac{1}{5}$$

204. Solution

```
Select[Permutations[Range[0, 9], {7}],
 Sqrt[FromDigits[{#[[1]], #[[2]], #[[3]], #[[3]], #[[4]], #[[5]], #[[6]]}]] ==
   FromDigits[{#[[7]], #[[4]], #[[3]], #[[3]]}] &]
```

{{4, 1, 3, 0, 8, 9, 2}}

205. Solution

```
N[Integrate[x^x, {x, 1, 100}]]
```

1.78464×10^{199}

206. Solution

```
FullSimplify[a^4 + b^4 + c^4 /.
  Solve[a + b + c == 3 && a^2 + b^2 + c^2 == 5 && a^3 + b^3 + c^3 == 7, {a, b, c}]]
```

{9, 9, 9, 9, 9, 9}

207. Solution

```
Solve[a + b + c == 3 && a^3 + b^3 + c^3 == 3, {a, b, c}, Integers]
```

{{a → -5, b → 4, c → 4}, {a → 1, b → 1, c → 1},
 {a → 4, b → -5, c → 4}, {a → 4, b → 4, c → -5}}

208. Solution

```
Select[Range[1000, 9999], #*4 == FromDigits[Reverse[IntegerDigits[#]]] &]
```

{2178}

209. Solution

```
Solve[(360 + 3 x)^2 == FromDigits[{4, 9, 2, y, 0, 4}] && 0 ≤ y ≤ 9 && x ≥ 0,
 {x, y}, Integers]
```

{{x → 114, y → 8}}

210. Solution

```
FullSimplify[Cot[10 Degree] Cot[30 Degree] Cot[50 Degree] Cot[70 Degree]]
```

3

211. Solution

```
Last[Complement[ Range[1, 1995],
  #[[1]] & /@ Select[Flatten[Table[{i, j}, {i, 1, 1995}, {j, 1, i}], 1],
    Total[IntegerDigits[#[[2]]]] == #[[1]] - #[[2]] &]]]
```

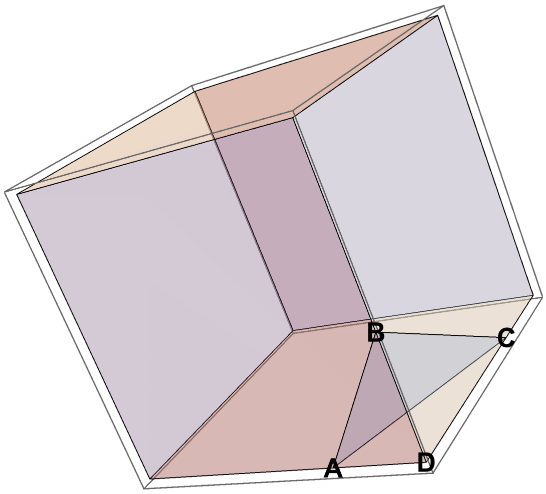
1985

212. Solution

```
Solve[Area[Triangle[{{x - 6, 0, 0}, {x, y, 0}, {x, 0, 8}}]] == 74 && y > 0, y, Reals]
```

{{y → 14}}

```
Graphics3D[{Opacity[0.3], Cuboid[{0, 0, 0}, {20, 20, 20}],
  Triangle[{{14, 0, 0}, {20, 14, 0}, {20, 0, 8}}], Opacity[1],
  Text[Style[#[[1]], Black, Bold, 14], #[[2]]] & /@
   {{"A", {14, 0, 0}}, {"C", {20, 14, 0}}, {"B", {20, 0, 8}}, {"D", {20, 0, 0}} }}]
```



213. Solution

```
Length[Solve[2 x y - 5 x + y == 55, {x, y}, Integers]]
```

16

214. Solution

```
Reduce[a^5 / ((a - b) (a - c) (a - d)) + b^5 / ((b - a) (b - c) (b - d)) +
    c^5 / ((c - a) (c - b) (c - d)) + d^5 / ((d - a) (d - b) (d - c)) == k &&
  a + b + c + d == 3 && a^2 + b^2 + c^2 + d^2 == 45, {a, b, c, d, k}]
```

$$\left(c == \frac{1}{2}\left(3 - a - b - \sqrt{81 + 6 a - 3 a^2 + 6 b - 2 a b - 3 b^2}\right) \,||\, c == \frac{1}{2}\left(3 - a - b + \sqrt{81 + 6 a - 3 a^2 + 6 b - 2 a b - 3 b^2}\right)\right) \&\& \; d == 3 - a - b - c \;\&\&\; k == 27$$

215. Solution

```
Solve[m^2 - n^2 == 1995 && m > 0 && n > 0, {m, n}, Integers]
```

{{m → 46, n → 11}, {m → 58, n → 37}, {m → 62, n → 43}, {m → 74, n → 59},
 {m → 146, n → 139}, {m → 202, n → 197}, {m → 334, n → 331}, {m → 998, n → 997}}

216. Solution

```
Solve[1 / m + 1 / n == 19 / 94, {m, n}, Integers]
```

```
{{m → 5, n → 470}, {m → 470, n → 5}}
```

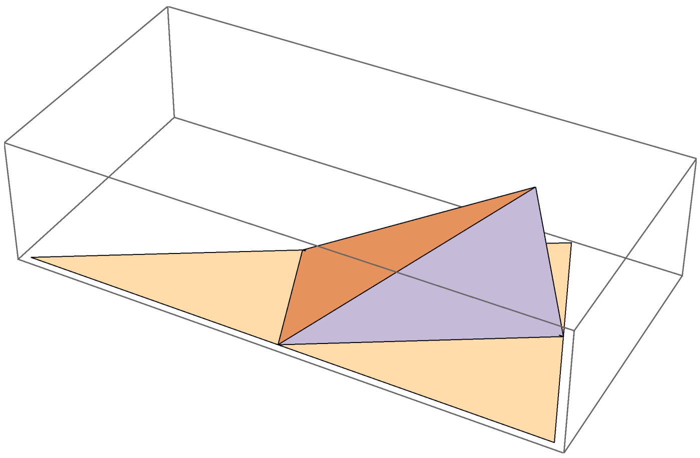
217. Solution

```
Solve[x^2 + y^2 + z^2 == k && 7 x^2 - 3 y^2 + 4 z^2 == 8 &&
  16 x^2 - 7 y^2 + 9 z^2 == -3 && x > 0 && y > 0 && z > 0, {x, y, z, k}, Integers]
```

```
{{x → 4, y → 10, z → 7, k → 165}}
```

218. Solution

```
Module[{P, Q, R, Tr, TrC, Tet},
 Tr = SSSTriangle[11, 20, 21];
 TrC = {#[[1]], #[[2]], 0} & /@ Tr[[1]];
 P = (TrC[[1]] + TrC[[2]]) / 2;
 Q = (TrC[[2]] + TrC[[3]]) / 2;
 R = (TrC[[3]] + TrC[[1]]) / 2;
 Tet = Tetrahedron[{P, Q, R, {x, y, z}}] /.
   Solve[EuclideanDistance[{x, y, z}, P] == 21 / 2 &&
      EuclideanDistance[{x, y, z}, Q] == 11 / 2 && EuclideanDistance[{x, y, z}, R] ==
       10 && x > 0 && y > 0 && z > 0, {x, y, z}, Reals] [[1]];
 Print[Show[Graphics3D[#] & /@ {Triangle[TrC], Triangle[{P, Q, R}], Tet}]];
 Volume[Tet]
]
```



```
45
```

219. Solution

```
Solve[(4 x^2 + 6 x + 4) (4 y^2 - 12 y + 25) == 28, {x, y}, Reals]
```

$$\left\{\left\{x \to -\frac{3}{4},\ y \to \frac{3}{2}\right\}\right\}$$

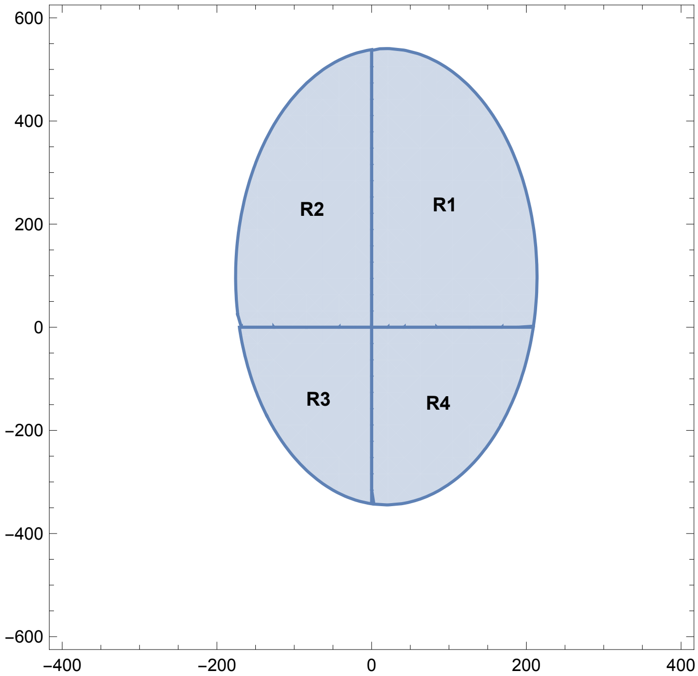
220. Solution

```
Solve[x + y + x y == 8 && y + z + y z == 15 && z + x + z x == 35 &&
  x + y + z + x y z == k && x > 0 && y > 0 && z > 0, {x, y, z, k}, Reals]
```

$$\left\{\left\{x \to \frac{7}{2},\ y \to 1,\ z \to 7,\ k \to 36\right\}\right\}$$

221. Solution

```
Module[{R1, R2, R3, R4, R},
 R = (x - 19)^2/19 + (y - 98)^2/98 <= 1998;
 R1 = ImplicitRegion[R && x ≥ 0 && y ≥ 0, {x, y}];
 R2 = ImplicitRegion[R && x ≤ 0 && y ≥ 0, {x, y}];
 R3 = ImplicitRegion[R && x ≤ 0 && y ≤ 0, {x, y}];
 R4 = ImplicitRegion[R && x ≥ 0 && y ≤ 0, {x, y}];
 Print[Show[RegionPlot[R1[[1]], {x, -400, 400}, {y, -600, 600}],
   RegionPlot[R2[[1]], {x, -400, 400}, {y, -600, 600}],
   RegionPlot[R3[[1]], {x, -400, 400}, {y, -600, 600}],
   RegionPlot[R4[[1]], {x, -400, 400}, {y, -600, 600}],
   Graphics[Text[Style["R1", Black, Bold, 10], RegionCentroid[R1]]],
   Graphics[Text[Style["R2", Black, Bold, 10], RegionCentroid[R2]]],
   Graphics[Text[Style["R3", Black, Bold, 10], RegionCentroid[R3]]],
   Graphics[Text[Style["R4", Black, Bold, 10], RegionCentroid[R4]]]
  ]];
 FullSimplify[
  RegionMeasure[R1] + RegionMeasure[R3] - RegionMeasure[R2] - RegionMeasure[R4]]]
```



7448

222. Solution

```
Take[IntegerDigits[Sum[i^i, {i, 1, 1000}]], 3]
```

{1, 0, 0}

223. Solution

```
Sqrt[1 + Sum[(4 i - 2)^3, {i, 1, 2001}]]
```

16 016 003

224. Solution

```
Factor[30 (a^2 + b^2 + c^2 + d^2) + 68 a b - 75 a c - 156 a d - 61 b c - 100 b d + 87 c d]
```

(3 a + 5 b - 6 c - 15 d) (10 a + 6 b - 5 c - 2 d)

225. Solution

```
Take[IntegerDigits[2^(2^24) + 1], -4]
```

```
{7, 5, 3, 7}
```

226. Solution

```
Total[Sqrt[1 + 1/#[[1]]^2 + 1/#[[2]]^2] & /@ Partition[Range[1, 2000], 2, 1]]
```

$$\frac{3\,999\,999}{2000}$$

227. Solution

```
Solve[1 / k + 1 / m + 1 / n == 19 / 84 && 0 < k < m < n , {m, k, n}, Integers]
```

```
{{m → 10, k → 8, n → 840}, {m → 12, k → 8, n → 56},
 {m → 13, k → 7, n → 156}, {m → 14, k → 7, n → 84}, {m → 15, k → 7, n → 60},
 {m → 16, k → 7, n → 48}, {m → 17, k → 6, n → 1428},
 {m → 18, k → 6, n → 252}, {m → 18, k → 7, n → 36}, {m → 20, k → 6, n → 105},
 {m → 20, k → 7, n → 30}, {m → 21, k → 6, n → 84}, {m → 21, k → 7, n → 28},
 {m → 24, k → 6, n → 56}, {m → 28, k → 6, n → 42}, {m → 39, k → 5, n → 1820},
 {m → 40, k → 5, n → 840}, {m → 42, k → 5, n → 420}, {m → 45, k → 5, n → 252},
 {m → 56, k → 5, n → 120}, {m → 60, k → 5, n → 105}, {m → 70, k → 5, n → 84}}
```

228. Solution

```
FullSimplify[Solve[ (Cos[3 x] / Cos[x] == 1 / 3) &&
    (Sin[3 x] / Sin[x]) == k && 0 ≤ x ≤ Pi/2, {x, k}, Reals] ]
```

$$\left\{\left\{x \to -2\,\text{ArcTan}\left[\sqrt{5} - \sqrt{6}\right],\ k \to \frac{7}{3}\right\}\right\}$$

229. Solution

```
FullSimplify[Times @@ (x^2 - 2 /. Solve[x^5 + x^2 + 1 == 0, x])]
```

```
-23
```

230. Solution

```
Module[{lf, p},
 lf = Sort[DeleteDuplicates[Flatten[Table[i/j, {i, 1, 98}, {j, i + 1, 99}]]]];
 p = Position[lf, 17 / 76][[1]];
 {lf[[p - 1]], lf[[p + 1]]}]
```

$$\left\{\left\{\frac{19}{85}\right\}, \left\{\frac{15}{67}\right\}\right\}$$

231. Solution

```
Select[Range[10 000, 99 999],
  Length[IntegerDigits[2 #]] == 5 && Sort[DeleteDuplicates[
       Join[IntegerDigits[#], IntegerDigits[2 #]]]] == Range[0, 9] &][[1]]
```

```
13 485
```

232. Solution

```
Solve[a11 + a12 + 2 + a14 == 14 &&
  a21 + 5 + a23 + a24 == 16 &&
  a31 + a32 + a33 + 8 == 26 &&
  3 + a42 + a43 + a44 == 30 &&
  a11 + a21 + a31 + 3 == 21 &&
  a12 + 5 + a32 + a42 == 25 &&
  2 + a23 + a33 + a43 == 13 &&
  a14 + a24 + 8 + a44 == 27 &&
  a11 + 5 + a33 + a44 == 20 &&
  3 + a32 + a23 + a14 == 16 && 1 ≤ a11 ≤ 9 && 1 ≤ a12 ≤ 9 &&
  1 ≤ a14 ≤ 9 && 1 ≤ a21 ≤ 9 && 1 ≤ a23 ≤ 9 && 1 ≤ a24 ≤ 9 && 1 ≤ a31 ≤ 9 &&
  1 ≤ a32 ≤ 9 && 1 ≤ a33 ≤ 9 && 1 ≤ a42 ≤ 9 && 1 ≤ a43 ≤ 9 && 1 ≤ a44 ≤ 9
    {a11, a12, a14, a21, a23, a24, a31, a32, a33, a42, a43, a44}, Integers]
```

{{a11 → 5, a12 → 3, a14 → 4, a21 → 4, a23 → 1,
 a24 → 6, a31 → 9, a32 → 8, a33 → 1, a42 → 9, a43 → 9, a44 → 9}}

```
Grid[{{0, 0, 2, 0, "s14"}, {0, 5, 0, 0, "s16"}, {0, 0, 0, 8, "s26"},
  {3, 0, 0, 0, "s30"}, {"s21", "s25", "s13", "s27"}}, Frame → All]
```

0	0	2	0	s14
0	5	0	0	s16
0	0	0	8	s26
3	0	0	0	s30
s21	s25	s13	s27	

233. Solution

```
FullSimplify[Solve[
  (3 x + y) (x + 3 y) Sqrt[x y] == 14 && (x + y) (x^2 + 14 x y + y^2) == 36 , {x, y}, Reals] ]
```

$\left\{\left\{x \to \frac{3}{2} - \sqrt{2},\ y \to \frac{3}{2} + \sqrt{2}\right\},\ \left\{x \to \frac{3}{2} + \sqrt{2},\ y \to \frac{3}{2} - \sqrt{2}\right\}\right\}$

234. Solution

```
Minimize[Max[a^2 + b, b^2 + a], {a, b}]
```

$\left\{-\frac{1}{4},\ \left\{a \to -\frac{1}{2},\ b \to -\frac{1}{2}\right\}\right\}$

235. Solution

$\text{FullSimplify}\left[\sqrt[3]{20 + 14\sqrt{2}} + \sqrt[3]{20 - 14\sqrt{2}}\right]$

4

236. Solution

$\text{Maximize}\left[\left\{\sqrt[4]{r} - \frac{1}{\sqrt[4]{r}},\ \sqrt[6]{r} + \frac{1}{\sqrt[6]{r}} == 6,\ r \geq 0\right\},\ r,\ \text{Reals}\right]$

$\left\{\frac{-1 + \left(3 + 2\sqrt{2}\right)^3}{\left(3 + 2\sqrt{2}\right)^{3/2}},\ \left\{r \to \left(3 + 2\sqrt{2}\right)^6\right\}\right\}$

237. Solution

```
Simplify[Det[{{(x^2 + 1)^2, (x y + 1)^2, (x z + 1)^2},
    {(x y + 1)^2, (y^2 + 1)^2, (y z + 1)^2},
    {(x z + 1)^2, (y z + 1)^2, (z^2 + 1)^2}}]]
```

$2\,(x - y)^2\,(x - z)^2\,(y - z)^2$

238. Solution

```
IrreduciblePolynomialQ[x^101 + 100 x^100 + 102]
```

False

239. Solution

```
FullSimplify[Total[ArcTan[x] /. Solve[x^3 - 10 x + 11 == 0, x, Reals]]]
```

$\frac{\pi}{4}$

240. Solution

$$\text{Solve}\left[\frac{4 x^2}{4 x^2 + 1} == y \;\&\&\; \frac{4 y^2}{4 y^2 + 1} == z \;\&\&\; \frac{4 z^2}{4 z^2 + 1} == x, \{x, y, z\}, \text{Reals}\right]$$

$\left\{\{x \to 0, y \to 0, z \to 0\}, \left\{x \to \frac{1}{2}, y \to \frac{1}{2}, z \to \frac{1}{2}\right\}\right\}$

241. Solution

```
RSolve[(n + 1) (n + 2) x[n] == 4 (n + 1) (n + 3) x[n - 1] - 4 (n + 2) (n + 3) x[n - 2] &&
   x[0] == 3 && x[1] == 4, x[n], n]
```

$\left\{\left\{x[n] \to -2^{-1+n}\left(-6 + n + n^2\right)\right\}\right\}$

242. Solution

```
Limit[n^2 Integrate[x^(x + 1), {x, 0, 1 / n}], n → Infinity]
```

$\frac{1}{2}$

243. Solution

```
FullSimplify[Sum[(-1)^k Factorial[n - k] Factorial[n + k], {k, 0, n}]]
```

$\frac{1}{2}\left(\text{Gamma}[1 + n]^2 + \frac{(-1)^n \text{Gamma}[2 + 2 n]}{1 + n}\right)$

244. Solution

$$\text{FullSimplify}\left[\text{Minimize}\left[\frac{(x^2 - x + 1)\,\hat{}\,3}{x^6 - x^3 + 1}, x, \text{Reals}\right]\right]$$

$\left\{-3 + 2\sqrt{3}, \left\{x \to \text{Root}\left[1 - 2 \#1 - 2 \#1^3 + \#1^4 \&, 1\right]\right\}\right\}$

245. Solution

$$\text{FullSimplify}\left[\text{Integrate}\left[\frac{x + \text{Sin}[x] - \text{Cos}[x] - 1}{x + E\,\hat{}\,x + \text{Sin}[x]}, x\right]\right]$$

$x - \text{Log}\left[e^x + x + \text{Sin}[x]\right]$

246. Solution

$$\text{FullSimplify}\left[\text{Integrate}\left[\frac{x\,\hat{}\,4 + 1}{x\,\hat{}\,6 + 1}, x\right]\right]$$

$\frac{1}{3}\left(2\,\text{ArcTan}[x] + \text{ArcTan}\left[\frac{x}{1 - x^2}\right]\right)$

247. Solution

```
FullSimplify[Integrate[Sqrt[(E^x + 1)/(E^x - 1)], x]]
```

$$\left(\text{ArcCosh}[e^x] - \text{ArcTan}\left[\frac{1}{\sqrt{-1+e^x}\,\sqrt{1+e^x}}\right]\right)\sqrt{\text{Coth}\left[\frac{x}{2}\right]}\,\sqrt{\text{Tanh}\left[\frac{x}{2}\right]}$$

248. Solution

```
FullSimplify[Integrate[(x^2 + 1)/(x^4 - x^2 + 1), x]]
```

$$\text{ArcTan}\left[\frac{x}{1-x^2}\right]$$

249. Solution

```
Integrate[(1 + 2 x^2) E^(x^2), x]
```

$$e^{x^2}\,x$$

250. Solution

```
Minimize[x^4 + 6 x^2 y^2 + y^4 - (9 / 4) x - (7 / 4) y, {x, y}]
```

$$\left\{-\frac{51}{32}, \left\{x \to \frac{3}{4}, y \to \frac{1}{4}\right\}\right\}$$

251. Solution

```
FullSimplify[Sin[70 Degree] Cos[50 Degree] + Sin[260 Degree] Cos[280 Degree]]
```

$$\frac{\sqrt{3}}{4}$$

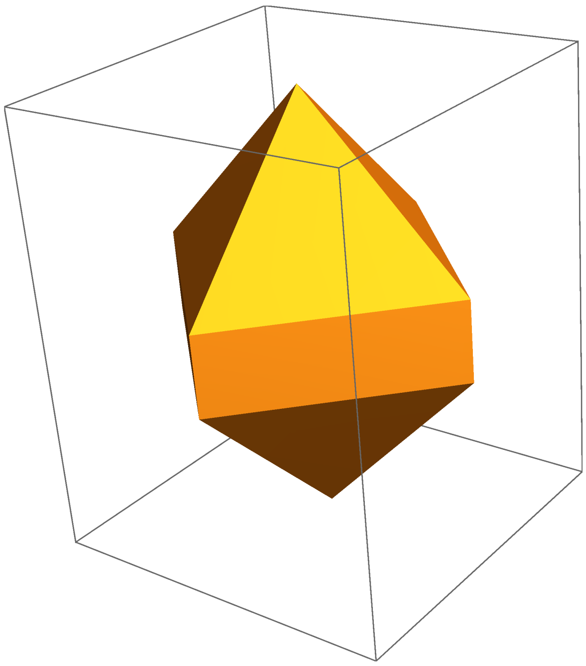
252. Solution

```
Solve[x^3 - 3 x == Sqrt[x + 2], x, Reals]
```

$$\left\{\{x \to 2\}, \left\{x \to \frac{1}{2}\left(-1-\sqrt{5}\right)\right\}, \left\{x \to \text{Root}\left[-1 - 2\,\#1 + \#1^2 + \#1^3\ \&,\ 2\right]\right\}\right\}$$

253. Solution

```
Module[{R},
 R = ImplicitRegion[Min[
      (x - 1 / 2) ^2 + (y - 1 / 2) ^2 + (z - 1 / 2) ^2,
      (x + 1 / 2) ^2 + (y + 1 / 2) ^2 + (z + 1 / 2) ^2,
      (x + 1 / 2) ^2 + (y + 1 / 2) ^2 + (z - 1 / 2) ^2,
      (x + 1 / 2) ^2 + (y - 1 / 2) ^2 + (z - 1 / 2) ^2,
      (x - 1 / 2) ^2 + (y + 1 / 2) ^2 + (z + 1 / 2) ^2,
      (x - 1 / 2) ^2 + (y - 1 / 2) ^2 + (z + 1 / 2) ^2,
      (x - 1 / 2) ^2 + (y + 1 / 2) ^2 + (z - 1 / 2) ^2,
      (x + 1 / 2) ^2 + (y - 1 / 2) ^2 + (z + 1 / 2) ^2] ≥ x^2 + y^2 + z^2, {x, y, z}];
 Print[RegionPlot3D[R]];
 RegionMeasure[R]
]
```



$\frac{9}{16}$

254. Solution

```
Solve[2 ∛(2 y - 1) == y^3 + 1, y, Reals]
```

$\{\{y \to 1\}, \{y \to \frac{1}{2}(-1 + \sqrt{5})\}\}$

255. Solution

```
Length[IntegerDigits[125^100]]
```

210

256. Solution

```
Solve[y (x + y)^2 == 9 && y (x^3 - y^3) == 7, {x, y}, Reals]
```

{{x → 2, y → 1}}

257. Solution

```
Solve[x^4 - 14 x^3 + 66 x^2 - 115 x + 265 / 4 == 0, x]
```

$$\left\{\left\{x \to \left(\frac{7}{2}-\frac{i}{2}\right)-\sqrt{4-2\,i}\right\},\ \left\{x \to \left(\frac{7}{2}-\frac{i}{2}\right)+\sqrt{4-2\,i}\right\},\ \left\{x \to \left(\frac{7}{2}+\frac{i}{2}\right)-\sqrt{4+2\,i}\right\},\ \left\{x \to \left(\frac{7}{2}+\frac{i}{2}\right)+\sqrt{4+2\,i}\right\}\right\}$$

258. Solution

```
Total[
 Select[Divisors[Times @@ (#[[1]]^#[[2]] & /@ Select[FactorInteger[19^88 - 1],
      #[[1]] == 2 || #[[1]] == 3 &] )], Mod[#, 2] == 0 && Mod[#, 3] == 0 &]]
```

744

259. Solution

```
Solve[x^2 + y^2 + 2 x y / (x + y) == 1 && Sqrt[x + y] ==  x^2 - y, {x, y}, Reals]
```

{{x → -2, y → 3}, {x → 1, y → 0}}

260. Solution

```
Solve[x^2 + y^2 + z^2 == 2 && x + y + z ==  2 + x y z, {x, y, z}, Reals]
```

{{x → 0, y → 1, z → 1}, {x → 1, y → 0, z → 1}, {x → 1, y → 1, z → 0}}

261. Solution

```
Total[Length[Divisors[#]] & /@ Range[1, 1989]]
```

15 422

262. Solution

```
FullSimplify[Sum[(-1)^(k - 1) / Binomial[2 n, k], {k, 1, 2 n - 1}], n ∈ Integers]
```

$$\frac{1}{1+n}$$

263. Solution

```
Module[{n},
 n = FromDigits[Fold[Join[#1, #2] &, (IntegerDigits[#] & /@ Range[19, 93])]];
 Max @@ (Select[Range[1, 2000], Mod[n, 3^#] == 0 &])]
```

1

264. Solution

```
Max @@ (#[[1]] & /@ Select[FactorInteger[Binomial[2000, 1000]], #[[1]] < 1000 &])
```

661

265. Solution

```
Solve[(x y - 7)^2 == x^2 + y^2 && x ≥ 0 && y ≥ 0, {x, y}, Integers]
```

$$\left\{\left\{y \to \text{ConditionalExpression}\left[\frac{7x}{-1+x^2} + \frac{\sqrt{49 - x^2 + x^4}}{\text{Abs}\left[-1+x^2\right]},\ (x \mid y) \in \text{Integers} \,\&\&\, x \ge 8\right]\right\},\right.$$

```
 {x → ConditionalExpression[0, (x | y) ∈ Integers],
  y → ConditionalExpression[7, (x | y) ∈ Integers]},
 {x → ConditionalExpression[3, (x | y) ∈ Integers],
  y → ConditionalExpression[4, (x | y) ∈ Integers]},
 {x → ConditionalExpression[4, (x | y) ∈ Integers],
  y → ConditionalExpression[3, (x | y) ∈ Integers]},
 {x → ConditionalExpression[7, (x | y) ∈ Integers],
  y → ConditionalExpression[0, (x | y) ∈ Integers]}}
```

266. Solution

```
Mod[2^1990, 1990]
```

1024

267. Solution

```
Length[Select[Subsets[Range[1, 300], {3}], Mod[Total[#], 3] == 0 &] ]
```

1 485 100

268. Solution

```
Length[Select[Subsets[Range[1, 20], {3}], Mod[Times @@ #, 4] == 0 &] ]
```

795

269. Solution

```
Module[{A, B},
 A = Select[Range[1, 700], Mod[#, 3] == 0 &];
 B = Select[Range[1, 300], Mod[#, 7] == 0 &];
 Length[Select[Tuples[{A, B}], #[[1]] ≠ #[[2]] && EvenQ[#[[1]] + #[[2]]] &]]]
```

4879

270. Solution

```
GCD @@ Select[Range[100 000, 999 999],
  EvenQ[#] && Sort[IntegerDigits[#]] == {1, 2, 3, 4, 5, 6} &]
```

6

271. Solution

```
Solve[(x + 1) (x^2 + 1) (x^3 + 1) == 30 x^3, x, Reals]
```

$$\left\{\left\{x \to \frac{1}{2}\left(3 - \sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(3 + \sqrt{5}\right)\right\}\right\}$$

272. Solution

```
Sum[1 / ((2 k - 1) (2 k + 1)), {k, 1, Infinity}]
Sum[1 / (k (k + 1) (k + 2)), {k, 1, Infinity}]
```

273. Solution

```
Factor[(x + y + z)^5 - x^5 - y^5 - z^5]
```

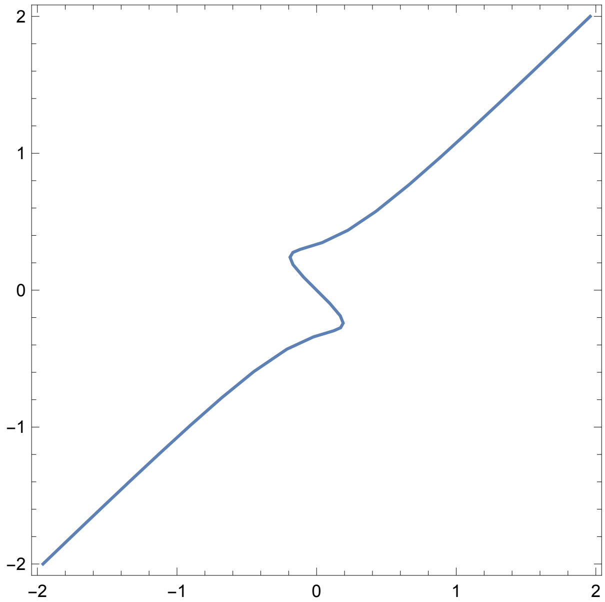
274. Solution

```
Solve[Total [#] == 0 & /@ Join[Partition[Table[x_i, {i, 1, 100}], 3, 1],
   {{x99, x100, x1}, {x100, x1, x2}}], Table[x_i, {i, 1, 100}], Reals]
```

$\{\{x_1 \to 0, x_2 \to 0, x_3 \to 0, x_4 \to 0, x_5 \to 0, x_6 \to 0, x_7 \to 0, x_8 \to 0, x_9 \to 0, x_{10} \to 0, x_{11} \to 0, x_{12} \to 0, x_{13} \to 0, x_{14} \to 0, x_{15} \to 0, x_{16} \to 0, x_{17} \to 0, x_{18} \to 0, x_{19} \to 0, x_{20} \to 0, x_{21} \to 0, x_{22} \to 0, x_{23} \to 0, x_{24} \to 0, x_{25} \to 0, x_{26} \to 0, x_{27} \to 0, x_{28} \to 0, x_{29} \to 0, x_{30} \to 0, x_{31} \to 0, x_{32} \to 0, x_{33} \to 0, x_{34} \to 0, x_{35} \to 0, x_{36} \to 0, x_{37} \to 0, x_{38} \to 0, x_{39} \to 0, x_{40} \to 0, x_{41} \to 0, x_{42} \to 0, x_{43} \to 0, x_{44} \to 0, x_{45} \to 0, x_{46} \to 0, x_{47} \to 0, x_{48} \to 0, x_{49} \to 0, x_{50} \to 0, x_{51} \to 0, x_{52} \to 0, x_{53} \to 0, x_{54} \to 0, x_{55} \to 0, x_{56} \to 0, x_{57} \to 0, x_{58} \to 0, x_{59} \to 0, x_{60} \to 0, x_{61} \to 0, x_{62} \to 0, x_{63} \to 0, x_{64} \to 0, x_{65} \to 0, x_{66} \to 0, x_{67} \to 0, x_{68} \to 0, x_{69} \to 0, x_{70} \to 0, x_{71} \to 0, x_{72} \to 0, x_{73} \to 0, x_{74} \to 0, x_{75} \to 0, x_{76} \to 0, x_{77} \to 0, x_{78} \to 0, x_{79} \to 0, x_{80} \to 0, x_{81} \to 0, x_{82} \to 0, x_{83} \to 0, x_{84} \to 0, x_{85} \to 0, x_{86} \to 0, x_{87} \to 0, x_{88} \to 0, x_{89} \to 0, x_{90} \to 0, x_{91} \to 0, x_{92} \to 0, x_{93} \to 0, x_{94} \to 0, x_{95} \to 0, x_{96} \to 0, x_{97} \to 0, x_{98} \to 0, x_{99} \to 0, x_{100} \to 0\}\}$

275. Solution

```
RegionPlot[ImplicitRegion[(2 x + 3 y) ^2 (y - x) == x + y, {{x, -2, 2}, {y, -2, 2}}]]
```



276. Solution

```
FullSimplify[RSolve[{a[n + 1] == a[n] / (1 + n a[n]), a[0] == A}, a[n], n] /. n → 1990]
```

$\{\{a[1990] \to \frac{A}{1 + 1\,979\,055\,A}\}\}$

277. Solution

```
Module[{n, x, A, B, C},
 x = IntegerDigits[Select[Range[100, 999], Module[{a, b, c, l},
       l = IntegerDigits[#];
       a = l[[1]];
       b = l[[2]];
       c = l[[3]];
       a > b > c &&
        Sort[IntegerDigits[FromDigits[{a, b, c}] * FromDigits[{b, c, a}] *
            FromDigits[{c, a, b}]]] == {2, 2, 2, 3, 3, 4, 5, 6, 8}] &][[1]]];
 A = x[[1]];
 B = x[[2]];
 C = x[[3]];
 FromDigits[{A, B, C}] * FromDigits[{B, C, A}] * FromDigits[{C, A, B}]
]
```

328 245 326

278. Solution

```
Maximize[{1 / (1 + x) + 1 / (2 - x), 0 ≤ x ≤ 1}, x, Reals]
```

$\left\{\frac{3}{2}, \{x \to 0\}\right\}$

279. Solution

```
Length[Select[Range[10 000, 99 999], Mod[#, 3] == 0 && Last[IntegerDigits[#]] == 6 &]]
```

3000

280. Solution

```
FullSimplify[Sum[k^2 / 2^k, {k, 1, n}]]
```

$6 - 2^{-n} (6 + n (4 + n))$

281. Solution

```
Integrate[Sin[x]^2 / x^2, {x, 0, Infinity}]
```

$\frac{\pi}{2}$

282. Solution

```
Factor[
 Det[{{1, a, a^2, a^4}, {1, b, b^2, b^4}, {1, c, c^2, c^4}, {1, d, d^2, d^4}}]]
```

$(a - b) (a - c) (b - c) (a - d) (b - d) (c - d) (a + b + c + d)$

283. Solution

```
FullSimplify[4 ArcTan[1 / 5] - ArcTan[1 / 239]]
```

$\frac{\pi}{4}$

284. Solution

```
Sum[Cos[k Pi / (2 n + 1)]^4, {k, 1, n}]
```

$\frac{1}{16} (-5 + 6 n)$

285. Solution

```
Solve[x^2 + y^2 + z^2 == 9 && x^4 + y^4 + z^4 == 33 && x y z  ==  -4, {x, y, z}, Reals]
```

```
{{x → -2, y → -2, z → -1}, {x → -2, y → -1, z → -2}, {x → -2, y → 1, z → 2},
 {x → -2, y → 2, z → 1}, {x → -1, y → -2, z → -2}, {x → -1, y → 2, z → 2},
 {x → 1, y → -2, z → 2}, {x → 1, y → 2, z → -2}, {x → 2, y → -2, z → 1},
 {x → 2, y → -1, z → 2}, {x → 2, y → 1, z → -2}, {x → 2, y → 2, z → -1}}
```

286. Solution

```
FullSimplify[
 α + β /. Solve[α^3 - 3 α ^2 + 5 α - 17 == 0 && β^3 - 3 β^2 + 5 β + 11 == 0 , {α, β}, Reals]]
```

```
{2}
```

287. Solution

```
Product[2^(n/2^n), {n, 1, Infinity}]
```

```
4
```

288. Solution

```
Solve[1 + 1996 x + 1998 y  ==  x y, {x, y}, Integers]
```

```
{{x → -3 986 011, y → 1995}, {x → 1, y → -1}, {x → 1997, y → -3 986 013},
 {x → 1999, y → 3 990 005}, {x → 3995, y → 3993}, {x → 3 990 007, y → 1997}}
```

289. Solution

```
Factor[(n^2)^2014 - (n^11)^106]
```

$$(-1+n)\, n^{1166}\, (1+n)\, \left(1-n+n^2\right) \left(1+n+n^2\right)$$

$$\left(1-n^3+n^6\right) \left(1+n^3+n^6\right) \left(1-n^9+n^{18}\right) \left(1+n^9+n^{18}\right)$$

$$\left(1-n+n^2-n^3+n^4-n^5+n^6-n^7+n^8-n^9+n^{10}-n^{11}+n^{12}-n^{13}+n^{14}-n^{15}+n^{16}-n^{17}+n^{18}-n^{19}+\right.$$
$$n^{20}-n^{21}+n^{22}-n^{23}+n^{24}-n^{25}+n^{26}-n^{27}+n^{28}-n^{29}+n^{30}-n^{31}+n^{32}-n^{33}+n^{34}-n^{35}+n^{36}-$$
$$\left.n^{37}+n^{38}-n^{39}+n^{40}-n^{41}+n^{42}-n^{43}+n^{44}-n^{45}+n^{46}-n^{47}+n^{48}-n^{49}+n^{50}-n^{51}+n^{52}\right)$$

$$\left(1+n+n^2+n^3+n^4+n^5+n^6+n^7+n^8+n^9+n^{10}+n^{11}+n^{12}+n^{13}+n^{14}+n^{15}+n^{16}+n^{17}+n^{18}+\right.$$
$$n^{19}+n^{20}+n^{21}+n^{22}+n^{23}+n^{24}+n^{25}+n^{26}+n^{27}+n^{28}+n^{29}+n^{30}+n^{31}+n^{32}+n^{33}+n^{34}+n^{35}+$$
$$\left.n^{36}+n^{37}+n^{38}+n^{39}+n^{40}+n^{41}+n^{42}+n^{43}+n^{44}+n^{45}+n^{46}+n^{47}+n^{48}+n^{49}+n^{50}+n^{51}+n^{52}\right)$$

$$\left(1-n+n^3-n^4+n^6-n^7+n^9-n^{10}+n^{12}-n^{13}+n^{15}-n^{16}+n^{18}-n^{19}+n^{21}-n^{22}+n^{24}-n^{25}+n^{27}-\right.$$
$$n^{28}+n^{30}-n^{31}+n^{33}-n^{34}+n^{36}-n^{37}+n^{39}-n^{40}+n^{42}-n^{43}+n^{45}-n^{46}+n^{48}-n^{49}+n^{51}-n^{52}+n^{53}-$$
$$n^{55}+n^{56}-n^{58}+n^{59}-n^{61}+n^{62}-n^{64}+n^{65}-n^{67}+n^{68}-n^{70}+n^{71}-n^{73}+n^{74}-n^{76}+n^{77}-n^{79}+$$
$$\left.n^{80}-n^{82}+n^{83}-n^{85}+n^{86}-n^{88}+n^{89}-n^{91}+n^{92}-n^{94}+n^{95}-n^{97}+n^{98}-n^{100}+n^{101}-n^{103}+n^{104}\right)$$

$$\left(1+n-n^3-n^4+n^6+n^7-n^9-n^{10}+n^{12}+n^{13}-n^{15}-n^{16}+n^{18}+n^{19}-n^{21}-n^{22}+n^{24}+n^{25}-\right.$$
$$n^{27}-n^{28}+n^{30}+n^{31}-n^{33}-n^{34}+n^{36}+n^{37}-n^{39}-n^{40}+n^{42}+n^{43}-n^{45}-n^{46}+n^{48}+n^{49}-n^{51}-n^{52}-$$
$$n^{53}+n^{55}+n^{56}-n^{58}-n^{59}+n^{61}+n^{62}-n^{64}-n^{65}+n^{67}+n^{68}-n^{70}-n^{71}+n^{73}+n^{74}-n^{76}-n^{77}+n^{79}+$$
$$\left.n^{80}-n^{82}-n^{83}+n^{85}+n^{86}-n^{88}-n^{89}+n^{91}+n^{92}-n^{94}-n^{95}+n^{97}+n^{98}-n^{100}-n^{101}+n^{103}+n^{104}\right)$$

$$\left(1-n^3+n^9-n^{12}+n^{18}-n^{21}+n^{27}-n^{30}+n^{36}-n^{39}+n^{45}-n^{48}+n^{54}-n^{57}+n^{63}-n^{66}+n^{72}-\right.$$
$$n^{75}+n^{81}-n^{84}+n^{90}-n^{93}+n^{99}-n^{102}+n^{108}-n^{111}+n^{117}-n^{120}+n^{126}-n^{129}+n^{135}-$$
$$n^{138}+n^{144}-n^{147}+n^{153}-n^{156}+n^{159}-n^{165}+n^{168}-n^{174}+n^{177}-n^{183}+n^{186}-n^{192}+n^{195}-$$
$$n^{201}+n^{204}-n^{210}+n^{213}-n^{219}+n^{222}-n^{228}+n^{231}-n^{237}+n^{240}-n^{246}+n^{249}-n^{255}+$$
$$\left.n^{258}-n^{264}+n^{267}-n^{273}+n^{276}-n^{282}+n^{285}-n^{291}+n^{294}-n^{300}+n^{303}-n^{309}+n^{312}\right)$$

$$\left(1+n^3-n^9-n^{12}+n^{18}+n^{21}-n^{27}-n^{30}+n^{36}+n^{39}-n^{45}-n^{48}+n^{54}+n^{57}-n^{63}-n^{66}+n^{72}+\right.$$
$$n^{75}-n^{81}-n^{84}+n^{90}+n^{93}-n^{99}-n^{102}+n^{108}+n^{111}-n^{117}-n^{120}+n^{126}+n^{129}-n^{135}-$$
$$n^{138}+n^{144}+n^{147}-n^{153}-n^{156}-n^{159}+n^{165}+n^{168}-n^{174}-n^{177}+n^{183}+n^{186}-n^{192}-n^{195}+$$
$$n^{201}+n^{204}-n^{210}-n^{213}+n^{219}+n^{222}-n^{228}-n^{231}+n^{237}+n^{240}-n^{246}-n^{249}+n^{255}+$$
$$\left.n^{258}-n^{264}-n^{267}+n^{273}+n^{276}-n^{282}-n^{285}+n^{291}+n^{294}-n^{300}-n^{303}+n^{309}+n^{312}\right)$$

$$\left(1-n^9+n^{27}-n^{36}+n^{54}-n^{63}+n^{81}-n^{90}+n^{108}-n^{117}+n^{135}-n^{144}+n^{162}-n^{171}+n^{189}-\right.$$
$$n^{198}+n^{216}-n^{225}+n^{243}-n^{252}+n^{270}-n^{279}+n^{297}-n^{306}+n^{324}-n^{333}+n^{351}-n^{360}+n^{378}-$$
$$n^{387}+n^{405}-n^{414}+n^{432}-n^{441}+n^{459}-n^{468}+n^{477}-n^{495}+n^{504}-n^{522}+n^{531}-n^{549}+n^{558}-$$
$$n^{576}+n^{585}-n^{603}+n^{612}-n^{630}+n^{639}-n^{657}+n^{666}-n^{684}+n^{693}-n^{711}+n^{720}-n^{738}+n^{747}-$$
$$\left.n^{765}+n^{774}-n^{792}+n^{801}-n^{819}+n^{828}-n^{846}+n^{855}-n^{873}+n^{882}-n^{900}+n^{909}-n^{927}+n^{936}\right)$$

$$\left(1+n^9-n^{27}-n^{36}+n^{54}+n^{63}-n^{81}-n^{90}+n^{108}+n^{117}-n^{135}-n^{144}+n^{162}+n^{171}-n^{189}-\right.$$
$$n^{198}+n^{216}+n^{225}-n^{243}-n^{252}+n^{270}+n^{279}-n^{297}-n^{306}+n^{324}+n^{333}-n^{351}-n^{360}+n^{378}+$$
$$n^{387}-n^{405}-n^{414}+n^{432}+n^{441}-n^{459}-n^{468}-n^{477}+n^{495}+n^{504}-n^{522}-n^{531}+n^{549}+n^{558}-$$
$$n^{576}-n^{585}+n^{603}+n^{612}-n^{630}-n^{639}+n^{657}+n^{666}-n^{684}-n^{693}+n^{711}+n^{720}-n^{738}-n^{747}+$$
$$\left.n^{765}+n^{774}-n^{792}-n^{801}+n^{819}+n^{828}-n^{846}-n^{855}+n^{873}+n^{882}-n^{900}-n^{909}+n^{927}+n^{936}\right)$$

290. Solution

```
PowersRepresentations[2010, #, 2] & /@ Range[1, 3]
```

```
{{}, {}, {{1, 28, 35}, {4, 25, 37}, {5, 7, 44},
  {5, 31, 32}, {7, 19, 40}, {11, 17, 40}, {16, 23, 35}, {19, 25, 32}}}
```

291. Solution

```
GCD @@ (# Product[k # + 1, {k, 1, 10}] & /@ Range[1, 2009])
```

```
2310
```

292. Solution

```
Factor[n^8 + n + 1]
```

$$\left(1+n+n^2\right) \left(1-n^2+n^3-n^5+n^6\right)$$

293. Solution

```
Solve[Product[(x - i), {i, 1, 6}] == 720, x, Integers]
{{x → 0}, {x → 7}}
```

294. Solution

```
Solve[(m^2 + n) (m + n^2) == (m + n)^3, {m, n}, Integers]
{{m → 0}, {n → 0}, {m → -5, n → 2}, {m → -1, n → 1}, {m → 1, n → -1},
 {m → 2, n → -5}, {m → 4, n → 11}, {m → 5, n → 7}, {m → 7, n → 5}, {m → 11, n → 4}}
```

295. Solution

```
Sum[Floor[√k] - Floor[∛k], {k, 1, 2003}]
40 842
```

296. Solution

```
FullSimplify[Product[Sin[k Pi / n], {k, 1, n - 1}]]
```

$2^{1-n}\, n$

297. Solution

```
Solve[# == 0 & /@
  CoefficientList[PolynomialMod[a x^17 + b x^16 + 1, x^2 - x - 1], x], {a, b}]
{{a → 987, b → -1597}}
```

298. Solution

```
Limit[(Sum[Binomial[3 n, 3 k], {k, 0, n}])^(1 / (3 n)), n → Infinity]
2
```

299. Solution

```
Solve[2 Log[x, c] - Log[c x, c] - 3 Log[c^2 x, c] == 0 && c > 0, x, Reals]
```

$\left\{\left\{x \to \text{ConditionalExpression}\left[c^{1/4}\, e^{-\frac{1}{4}\sqrt{33}\sqrt{\text{Log}[c]^2}},\ c > 0\right]\right\},\right.$
$\left.\left\{x \to \text{ConditionalExpression}\left[c^{1/4}\, e^{\frac{1}{4}\sqrt{33}\sqrt{\text{Log}[c]^2}},\ c > 0\right]\right\}\right\}$

300. Solution

```
Solve[3^(2 x) - 34 (15^(x - 1)) + 5^(2 x) == 0, x, Reals]
{{x → -1}, {x → 1}}
```

301. Solution

```
FullSimplify[Sum[2^(k - 1) Binomial[4 n - 2, 2 k], {k, 1, 2 n - 1}]]
```

$\frac{1}{4}\left(-2 + \left(3 - 2\sqrt{2}\right)\left(1 + \sqrt{2}\right)^{4n} + \left(1 - \sqrt{2}\right)^{4n}\left(3 + 2\sqrt{2}\right)\right)$

302. Solution

```
Solve[6 x^2 - 3 x y - 13 x + 5 y + 11 == 0, {x, y}, Integers]
{{x → 1, y → -2}, {x → 2, y → 9}}
```

303. Solution

```
Minimize[
  {((x + 1 / x) ^6 - (x^6 + 1/x^6) - 2) / ((x + 1 / x) ^3 + (x^3 + 1/x^3)), x > 0}, x]
{6, {x → 1}}
```